MEFHARET KOCATEPE AND HER CONTRIBUTION TO FUNCTIONAL ANALYSIS

ALEXANDER GONCHAROV

Mefharet Kocatepe (at that time she was Nuriye Mefharet Alpseymen) defended her PhD Thesis "Basic Sequences in Some Nuclear Köthe Sequence Spaces" in 1978 in the University of Michigan. Her thesis advisor was Professor Melapalayam S. Ramanujan. After this, she worked at the Middle East Technical University. In the mid-1980s, when Bilkent University was founded, she began working at the Department of Mathematics. She was the head of the department for over 20 years. Professor Kocatepe played a decisive role in creating the department in its current form.

The last years of her academic career, Professor Kocatepe was the Dean of the Faculty of Education. Her activities in this position lead to the improvement of teaching and learning in Bilkent.

Mefharet Kocatepe made a substantial contribution to the development of mathematics in Turkey. She was one of the organizers of a series of three conferences "Linear Topological Spaces and Spaces of Analytic Functions", which were held at the International Center for Physics and Applied Mathematics of Trakya University in Edirne in the second half of 90s. She was also in the International Advisor Board and in the Organizing Committee of the considerable international conference "Computational Methods and Function Theory", Bilkent, 2009.

The area of scientific interests of Professor Kocatepe can be called "Structure Theory of Nuclear Fréchet spaces". If such a space has a topological basis, then it can be represented as a Köthe space, which is exactly the space of coefficients of basic expansions of elements. In addition, in all known cases, if a nuclear Fréchet (NF) space has a topological basis, then this basis is unique (up to permutations of elements of the basis, multiplying by nonzero

coefficients, and isomorphic transformations). Two bases are called *quasi-equivalent* if one basis can be obtained from another using the above operations. Mityagin conjectured that any two bases in an NF space are quasi-equivalent.

In her first result, published in J. Reine Angew. Mat. (1975) Prof. Kocatepe confirmed the conjecture for the so-called G_∞ and G_1 spaces (with a basis). The spaces of entire functions $A(\mathbb{C})$ and analytic functions on the unit disc $A(\mathbb{D})$ are models for power series spaces of infinite and, accordingly, of finite type. Dragilev introduced important linear topological interpolating invariants D_1 and D_2 distinguishing, in particular, the power series spaces of different type. Also he proposed sequence spaces $L_f(a,r)$ generalizing power series spaces. The spaces G_∞ and G_1, in turn, generalize Dragilev's D_1 and D_2 spaces.

Later, Professor Kocatepe and her students returned to the problem of quasi-equivalence and considered the possibility of reducing this problem to the construction of a quasi-diagonal isomorphism.

On the other hand, there are nuclear Fréchet spaces without bases. As a result of this fact, a number of problems were posed. One of the main problems is to characterize all subspaces with a basis of a given NF space. In particular, Mityagin-Pełczyński problem is considered as one of the most important unsolved problems in the field: Suppose X is a nuclear Fréchet space with basis and E is a complemented subspace of X. Does E possess a basis?

Professor Kocatepe made an essential contribution in this direction. She proved the existence of bases in a wide class of subspaces of Dragilev spaces, as well as in Cartesian and tensor products of some Köthe spaces.

The next direction of her research is the isomorphic classification of NF spaces. Professor Kocatepe did a comprehensive characterization of Dragilev's spaces and found a surprising result: Dragilev's spaces $L_f(a,r)$ of finite and infinite type can be isomorphic. This disproved Dragilev's hypothesis.

By means of counting invariants (as the diametral dimension) as well as compound invariants, suggested by Zakharyuta, Professor Kocatepe with coauthors presented various families of continual cardinality of pairwise non-isomorphic spaces. Basically, these were classes of spaces of infinitely differentiable functions on a sharp cusp and their analogues of Whitney spaces for compact sets on the line. The interest in these spaces is due to the fact that they were (and remain!) possible candidates for a functional $NF-$ space *with natural topology* and without basis.

In particular, in Michigan Math. J. (1997), Professor Kocatepe et al. analyzed the spaces of Whitney functions defined on convergent to a point sequences of segments. Some geometric conditions of the extension property for such sets were obtained, that were very useful for future investigation of the existence of a continuous linear extension operator for Whitney spaces.

In Canad. J. Math. (2002), the concept of logarithmic dimension $\lambda_0(K)$ was introduced, as an analog of the Hausdorff dimension with respect to the function $\psi(r) = (\log \frac{1}{r})^{-1}$, corresponding to the logarithmic measure. The value $\lambda_0(K) = 1$ is critical for the extension problem, in Potential Theory and in some other areas.

Thus, we see that Professor Kocatepe dealt with really difficult tasks. The quasi-equivalence, Mityagin-Pełczyński problem, and the mentioned above basis problem remain open to our days. However, her results clarified many things. We are confident that her contribution is a solid base for future attacks on these challenges.

Bibliography

[1] Alpseymen, M.: A generalization of Dragilev's theorem. *J. Reine Angew. Math.* 276 (1975), 124–129.

[2] Alpseymen, Nuriye Mefharet: *Basic Sequences in some Nuclear Köthe Sequence Spaces*, Thesis (Ph.D.) — University of Michigan. 1978. 92 pp, ProQuest LLC.

[3] Dubinsky, E.; Alpseymen, M.: De Grande-De Kimpe, N. Basic sequences in some stable, nuclear $Lf(b,r)$-spaces. Nederl. Akad. Wetensch. *Indag. Math.* 41 (1979), no. 2, 203–215.

[4] Alpseymen, M.; Ramanujan, M. S.; Terzioğlu, T.: Subspaces of some nuclear sequence spaces. Nederl. Akad. Wetensch. *Indag. Math.* 41 (1979), no. 2, 217–224.

[5] Alpseymen, M.: Basic sequences in stable infinite type power series spaces. *Studia Math.* 70 (1981), no. 1, 21–26.

[6] Kocatepe, Mefharet: On Dragilev spaces and the functor Ext. *Arch. Math. (Basel)* 44 (1985), no. 5, 438–445.

[7] Kocatepe, Mefharet: Some counterexamples on Dragilev spaces. Proceedings of the functional analysis conference (Silivri/Istanbul, 1985). *Doğa Mat.* 10 (1986), no. 1, Special Issue, 136–142.

[8] Kocatepe, Mefharet An isomorphism theorem for Dragilev spaces. *Arch. Math. (Basel)* 50 (1988), no. 3, 281–286.

[9] Kocatepe, Mefharet: Classification of Dragilev spaces of types -1 and 0. *Math. Balkanica (N.S.)* 2 (1988), no. 2-3, 266–275.

[10] Kocatepe, M.; Nurlu, Z.: Some special Köthe spaces, in *Advances in the Theory of Fréchet Spaces (Istanbul, 1988)*, pp. 269–296, NATO Adv. Sci. Inst. Ser. C: Math. Phys. Sci., 287, Kluwer Acad. Publ., Dordrecht, 1989.

[11] Kocatepe, Mefharet: A note on vanishing of the functor Ext1 for Köthe spaces. *Manuscripta Math.* 71 (1991), no. 2, 113–119.

[12] Ertuğrul, Z.; Kocatepe, M:. On the existence of a pseudo-regular basis in some Köthe spaces. *Turkish J. Math.* 19 (1995), no. 1, 48–55.

[13] Kocatepe, Mefharet; Zahariuta, Viacheslav P.: Köthe spaces modeled on spaces of C^∞ functions. *Studia Math.* 121 (1996), no. 1, 1–14.

[14] Goncharov, Alexander P.; Kocatepe, Mefharet: Isomorphic classification of the spaces of Whitney functions. *Michigan Math.* J. 44 (1997), no. 3, 555–577.

[15] Arslan, Bora; Kocatepe, Mefharet: An application of linear topological invariants. *Turkish J. Math.* 21 (1997), no. 3, 343–356.

[16] Goncharov, Alexander P.; Kocatepe, Mefharet: A continuum of pairwise nonisomorphic spaces of Whitney functions on Cantor-type sets. Dedicated to Professor Vyacheslav Pavlovich Zahariuta. *Linear Topol. Spaces Complex Anal.* 3 (1997), 57–64.

[17] Arslan, Bora; Kocatepe, Mefharet: The quasi-equivalence problem for a class of Köthe spaces. *Proceedings of the Second International Workshop on Functional Analysis (Trier, 1997). Note Mat.* 17 (1997), 113–120 (1999).

[18] Arslan, Bora; Goncharov, Alexander P.; Kocatepe, Mefharet: Sequence space representations of spaces of Whitney functions. *Results Math.* 37 (2000), no. 1-2, 3–12.

[19] Arslan, Bora; Goncharov, Alexander P.; Kocatepe, Mefharet: Spaces of Whitney functions on Cantor-type sets. *Canad. J. Math.* 54 (2002), no. 2, 225–238.

[20] Kesir, Mustafa; Kocatepe, Mefharet: Existence of basis in some Whitney function spaces. *Results Math.* 48 (2005), no. 1-2, 89–96.