ABSTRACTS

Extremal Problems on Bergman Spaces A^1_α and Besov Spaces. Extremal problems in different function spaces have long been investigated. Ferguson provides a method, using Bergman projections, to solve certain types of the extremal problems in Bergman spaces for $1 < p < \infty$ in his work in 2004. Later the method is extended to weighted Bergman spaces for $1 < p < \infty$ by Özbek in 2017. Now, we extend this method to the $p = 1$ case. The two cases differ in the structure of Bergman projections and dual spaces. First, we define some function spaces, namely weighted Bergman spaces, the Bloch space, Besov spaces, and show the usage of Bergman projection on these space. Then, we use Bergman projection to find a candidate function for the solution in the $p = 1$ case, and we prove that the candidate function is the solution if it satisfies some conditions.

Alper Balcı, Bilkent University

The Local and the Global, in Finite Group Representation Theory. Usually, in finite group theory, the jargon "local" refers to p-subgroups and constructions related to p-subgroups, where p is a prime. The principle is that the p-subgroups of a finite group G, together with further information such as the G-conjugations between them, ought to determine the "local properties" of G. The jargon "local properties" is to be interpreted in such a way as to make the principle sound. Thus, passage to the "local" is a deliberate forgetting that which might be called "global", for instance, the given group G.

When considering some category of linear representations of G, we may pass to the Grothendieck module $F(G)$. Allowing G to vary, we can make the class of finite groups become a category in such a way that F becomes a functor. Via F, relationships between finite groups are transported to relationships between the modules $F(G)$. We can then pass from a study of the modules $F(G)$ to a study of the structure of F. That makes sense, because F can be viewed as a module of a suitable algebra.

We can then ask how much of the "global" structure of F derives from some suitably defined "local information". We can also ask how the "local" parts fit together. Ideally, the "local" parts would be related to p-groups. Then the outlook of local representation theory might, at last, fully make sense and be vindicated.

As matters stand, the above account may have stretched the words "local" and "global" too far. But the liberties have at least set up a pun: of course, the representation theory of finite groups is a vast international endeavour, nevertheless, the present conference does give me an opportunity to illustrate the so-called "functorial" approach to representation theory through some comments on the PhD work of five Bilkent students: Ergün Yaraneri, Olcay Coşkun, İpek Tuvay, Hatice Mutlu Akaturk, İsmail Alperen Öğüt, in chronological order.

Laurence Barker, Bilkent University

Isomorphic Classification of Nuclear Fréchet Spaces. The talk is an introduction to the problem of isomorphic classification of NF spaces. The main types of linear topological invariants are considered.

A brief review of the results of Professor Kocatepe in this area is given.

Alexandre Goncharov, Bilkent University

Diametral Dimension of some Infinite Products. Recently, a basis was constructed for the space of Whitney jets defined on a sequence with "moderate" rate of convergence. We believe these kinds of spaces are isomorphic to a cartesian product of infinitely many copies of a power series of infinite type and another power series space of infinite type. However, these power series spaces may not be stable. For this reason, we have investigated the linear topological structure of not necessarily stable power series spaces of infinite type and the infinite cartesian products of the same copy. Understanding the linear topological structure of the infinite products will shed light into why the moderateness condition had to be induced to construct a basis. In this talk, we present the diametral dimension of the cartesian product of infinitely many copies of power series spaces. We then give a sufficient condition for two of these spaces to be isomorphic.

Fehmi Ekin Giritlioğlu, Bilkent University

Loewner Theory and Stochastic Loewner Evolution or Schramm Loewner Evolution. Loewner theory is about the parametric representation of univalent functions on the unit disc in $\mathbb{C}$. This set of univalent functions are the solution set of certain differential equations, called Loewner differential equations and the solution set called Loewner set. Using this solution set Loewner succeeded in proving a special case of the Bieberbach conjecture $|a_3| \leq 3$, in 1923. Now it is known as Bieberbach-DeBranges theorem and solved by DeBranges in 1985. In fact we use two versions of the Loewner's equation:

1) Mapping the unit disk onto slit disc,
2) Mapping the slit half-plane onto half-plane.

Stochastic Loewner equation (SLE) introduced by Schramm, in the slit half-plane is the random collection of the conformal maps $g(z,t)$ obtained by solving the Loewner equation

$$\frac{\partial g(z,t)}{\partial t} = \frac{2}{g(z,t) - U(t)}, \quad g(z,0) = z, \tag{0.1}$$

by choosing $U(t) = \sqrt{\kappa} B_t$ where B_t is a standard one dimensional Brownian motion.

Azize Hayfavi, Middle East Technical University

On Well-posedness of IBV Problems for the Nonlinear Schrödinger Equation. In this talk, we consider the local well-posedness problem for a coupled system of nonlinear Schrödinger equations (NLS) associated with Dirichlet/Neumann/mixed type boundary conditions. We revisit the local wellposedness theory for classical NLS on the half line and review some initial-boundary type Strichartz estimates (e.g., [1]-[2], [4]-[5]). These include some new estimates for the Neumann case that we prove by using weak solution formulas obtained through Fokas's unified transform method [3] and traditional tools from oscillatory integral theory [6]. These estimates are crucial for treatment of low regularity solutions for which Banach algebra property fails. We also give a few remarks regarding the global well-posedness of the coupled system. This is joint work with T. Özsarı (Bilkent).

References

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Bilge Köksal, Bilkent University

On the Structure Theory of Nuclear Fréchet Spaces with Basis. After being defined by A. Grothendieck, nuclear spaces have become the focus of extensive research especially in 1970 and 1980's. In this talk we try to point out certain key developments in the theory. The emphasis is on the contribution of Professor Mefharet Alpseymen Kocatepe in whose honour this symposium is organized. Thus, the concrete Koethe spaces, especially $L_f(a,r)$, form the bulk of this summary.

Zafer Nurlu, Middle East Technical University

Pavlucki-Plesniak Operator for Non-Markov Sets. We discuss the extension property for the spaces of Whitney functions. We are interested in the Pavlutsky-Plesnyak extension operator, which is continuous for Markov compacta. We test its continuity on a model example of a non-Markov set.

Yaman Paksoy, Bilkent University

Achievements of our Geometry Group during the First Thirty Years of our Department, and a Little Bit of History. This is going to be mostly an expository talk on the first years of our department and the activities of our geometry group over the years. Here *Geometry* is used to mean *Algebraic Geometry*, and anyone who published an article with Mathematics Subject Classification Primary 14 is considered a member of our group. I will talk about the general panorama of the geometry group as well as the prizes and achievements of some of our group members.

Ali Sinan Sertöz, Bilkent University

Some Results on Bases in Whitney Spaces. This talk is mainly about two recent results by Goncharov and Şengül [1, 2] in relation with the conjecture of Mityagin about bases in nuclear Fréchet spaces. In order to understand the statements of the results, definitions of Whitney spaces, Cantor-type sets and bases in Whitney spaces for general Cantor-type sets are given. Some results in the literature supporting the conjecture are going to be reviewed and two theorems on bases in Whitney spaces and their quasi-equivalence will be obtained using some results by Arslan, Kocatepe and Goncharov in [3] and [4].

References

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Yasemin Şengül, Cardiff University

The Gaussian Contraction Theorem. In this talk we give a simple, dimension independent, proof of the Gaussian contraction theorem showing that all measures which have H-log concave densities with respect to the Gaussian measure can be realized with Lipschitz contractions.

Ali Süleyman Üstünel, Bilkent University

Finite Group Actions on Products of Spheres. Classifying all finite groups that can act freely on a sphere is a classical problem in algebraic topology which has led to many interesting mathematics. As a generalization of this problem there are many open problems and conjectures for finite group actions on a product of spheres. In this talk, I will give a short survey of the results that we obtained while working towards the solution of some of these problems with Özgün Ünlü and others during the last 20 years when Mefharet Kocatepe was the chair in our department.

One of these conjectures is the rank conjecture. The rank of a finite group G is defined to be the largest positive integer r such that $(\mathbb{Z}/p)^r \leq G$ for some prime p. The rank conjecture, due to Benson and Carlson, states that a finite group G acts freely on a finite CW-complex X homotopy equivalent to a product of k spheres if and only if $\operatorname{rk}(G) \leq k$. This conjecture is known to be true for $k = 1$ by classical Smith theory and by a theorem of Swan. More recently, it has been proved by Jackson, building on results by Adem and Smith, that the construction direction of the conjecture holds for every rank 2 finite group which does not involve the group $\operatorname{Qd}(p) = (\mathbb{Z}/p \times \mathbb{Z}/p) \rtimes SL_2(p)$ for any odd prime p. With Ünlü, we proved that every finite group acts freely on a product of k spheres for some k, with trivial action on homology.

Ergün Yalcın, Bilkent University

A Modular Equation of Degree 61. A modular equation of degree n is an equation that relates classical theta functions with arguments q and q^n. The theory of modular equations started with the works of Landen, Jacobi and Legendre. The theory gained popularity again with enormous contributions made by Ramanujan. In this talk we will give a brief introduction to the theory of modular equations and then obtain a new modular equation of degree 61 by using a generalization of a theta function identity due to David M. Bressoud. This is a joint work with Ahmet Güloğlu.

Hamza Yeşilyurt, Bilkent University