Multiple Large-Scale Coherent Structures in Free Turbulent Shear Flows

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1. Introduction

We apply ideas developed earlier [1], [2] to illustrate the possibilities of describing free shear layer development with multiple subharmonic frequency components, fine-grained turbulence and their effect on the spreading rate.

The heuristic description of shear layer development via multiple subharmonics was given by Ho [3]. On the basis of a two-frequency mode interaction [4]. we construct a multiple-subharmonic model in a growing free shear layer by postulating that only neighboring modes enter into a binary-frequency interaction. This relies on the observations of real, developing shear layers where higher frequency modes occur closer to the trailing edge of the splitter plate separating the two streams (for a given initial shear layer thickness), while lower frequency modes develop further downstream. In this case, binary frequency interactions are sufficient.

The issue of so-called "small scale transition" (see [5]) in the presence of a single mode was first addressed by Liu and Merkine [6]. The multiple mode problem brings in continued straining of the fine-grained turbulence by successive lower frequency modes, as the higher frequency modes decay, thus providing continued supply of energy from the coherent eddies (in addition from the mean motion).

In the following we state the nonlinear spatial evolution equaitons for the amplitude (or energy) of the coherent modes, the spreading rate of the mean motion and the energy of the fine-grained turbulence, derived using shape assumptions from the energy integral equations.

2. Summary of Nonlinear Interaction Equations

The fundamental energy density is denoted by A_f^2, where A_f is the fundamental amplitude. Its spatial development is described by

$$\bar{I}_f \frac{d\delta A_f^2}{dx} = I_{rsf}A_f^2 + I_{fs1}A_{s1}^2A_f - I_{wtf}EA_f^2 - \frac{1}{Re}I_{\phi f}A_f^2/\delta, \tag{1}$$

where $\bar{I}_f$ is the mean of an energy advection integral, I_{rsf} is the Reynolds stress energy transfter, I_{fs1} is the binary-frequency fundamental (f) and first subharmonic

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(s1) energy-exchange integral, I_{wtf} is the wave-fine-grained turbulence energy exchange integral, $I_{\phi f}$ is the fundamental viscous dissipation integral, δ is the mean shear layer thickness, A_{s1} is the first subharmonic amplitude, E is the fine-grained turbulence energy density and Re is the Reynolds number based on the initial shear layer thickness δ_0 and averaged two-stream velocity $\bar{U}_\infty$. We have not considered frequency components higher than the "fundamental", although such could be easily accommodated.

The n^{th} subharmonic amplitude equation would involve the binary-frequency connections with the neighboring higher and lower frequency components. The n^{th} subharmonic amplitude equation appears in the form

$$\bar{I}_{sn} \frac{d\delta A_{sn}^2}{dx} = I_{rssn} A_{sn}^2 - I_{s(n-1)sn} A_{s(n-1)}^2 A_{sn} + I_{sns(n+1)} A_{s(n+1)}^2 A_{sn} - I_{wtsn} E A_{sn}^2 - \frac{1}{Re} I_{\phi sn} A_{sn}^2/\delta, \tag{2}$$

where the integrals, with the appropriate subscripts, have corresponding meaning as those in (1).

The kinetic energy equation of the mean motion, upon a similarity shape assumption (which could be made more complicated), reduces to, except for a minus sign, the evolution equation for the shear layer thickness δ,

$$\bar{I} \frac{d\delta}{dx} = I_{rsf} A_f^2 + \sum_n I_{rssn} A_{sn}^2 + I'_{rs} E + \frac{1}{Re} I_\Phi/\delta, \tag{3}$$

where $\bar{I}$ is the mean kinetic energy advective integral, I_Φ the dissipation integral, and I'_{rs} the fine-grained turbulence energy production integral. It is obvious from (3) that the shear layer spreading rate is positive if energy is transferred from the mean flow to the coherent modes, turbulence or converted into heat. In this case, the negative energy transfer, or energy return to the mean motion from the damped disturbances contributes negatively to the spreading rate.

The kinetic energy balance for the fine-grained turbulence, with shape assumptions for the mean stresses, give

$$I_t \frac{d\delta E}{dx} = I'_{rs} E + E\left[A_f^2 I_{wtf} + \sum_n A_{sn}^2 I_{wtsn}\right] - I_{\phi'} E^{3/2}, \tag{4}$$

where I_t is the turbulence energy advection integral, $I_{\phi'}$ the dissipation integral.

We refer to more detailed discussions elsewhere [2] of the various aspects of the formulation similar to (1) - (4). Integrals involving the mean motion would reduce to constants with similarity shape assumptions. Integrals involving the wave modes are functions of the local shear layer thickness through the dependence of the local linear stability theory on the local, dimensionless frequency parameter $\beta = 2\pi f \delta / U_\infty$, where f is the frequency.

3. Results

The formulation of the spatially developing "wave envelope" equations of the two-dimensional coherent modes (1), (2) as well as that for the "mean motion" (3), (4) constitute an initial value problem. As such, it affords possibilities of parametric studies for flow control.

Within the limited length of the present description, we present new numerical results to illustrate the effect of changing the set of forcing frequencies on flow development and separately, the effect of initial turbulence level in the shear layer.

The parameter values follow closely those of Ho and Huang [7], the velocity ratio is R = 0.31, the most amplified frequency parameter is about $\beta_{f0} = 0.5916$. The successive initial subharmonic frequency parameter would be successively halved. The dimensionless initial energy levels (e.g., $E_f = \delta A_f^2$) are fixed as $E_{f0} = 2.62 \times 10^{-3}$, $E_{s10} = 4 \times 10^{-5}$, $E_{s20} = E_{s30} = 10^{-5}$. Subharmonics up to s3 are included for the region of interest up to $x/\delta_0 \approx 500$. The initial turbulence energy is taken to be $E_0 = 10^{-6}$, corresponding to essentially an initial laminar flow. The shape of the coherent eddies are described by the local linear inviscid stability theory (and the amplitude by the present nonlinear theory), and is used to estimate the viscous dissipation integral. The Reynolds number in the coefficient is here given the value Re = 115. On the other hand, in regions of large amplitude coherent structure, viscous dissipation play a minor role. The phase angles between the coherent modes are fixed at 180°, in which case, energy is transferred from higher to lower frequency modes. The shear layer thickness development is shown in Figure 1. The "standard" case, denoted by 1, is for $\beta_{f0} = 0.5916$ ($\beta_{s10} = \beta_{f0}/2$, and so on). Each step-like behavior is due to the coherent mode peak and decay in the respective step region. In this case, within the region of interest, the first three "steps" are due to f, s1 and s2, while s3 would peak and decay beyond the region shown. The fine-grained turbulence becomes "fully" developed as s1 is decaying and s2 is developing a peaking, thus giving the subsequent linear growth of the shear layer. These are a direct consequence of the physical interpretation of the shear layer spreading rate (3). Similar reasoning lead to the behavior of the shear layer growth for the following cases, in which the set of forcing frequencies is tuned higher (case 2: $\beta_{f0} = 0.8874$; case 3: $\beta_{f0} = 1.183$) and lower (case 4: $\beta_{f0} = 0.2958$; case 5: $\beta_{f0} = 0.4437$; case 6: $\beta_{f0} = 0.1479$).

In case 2, the f mode decays at the outset, the turbulence becomes fully developed as s2 decays and s3 is peaking. In case 6, the f mode has a long drawn out peak, thus accounting for the dominant spread in the shear layer shown. In this case, turbulence becomes fully developed as the f mode decays and the s1 mode reaches it peak. The "small-scale transition" is accomplished the earliest in case 4 ($x/\delta_0 \approx 200$) while latest in case 6 ($x/\delta_0 \approx 360$) for the set of given initial conditions. Of course, varying the initial conditions would allow further studies of control possibilities.

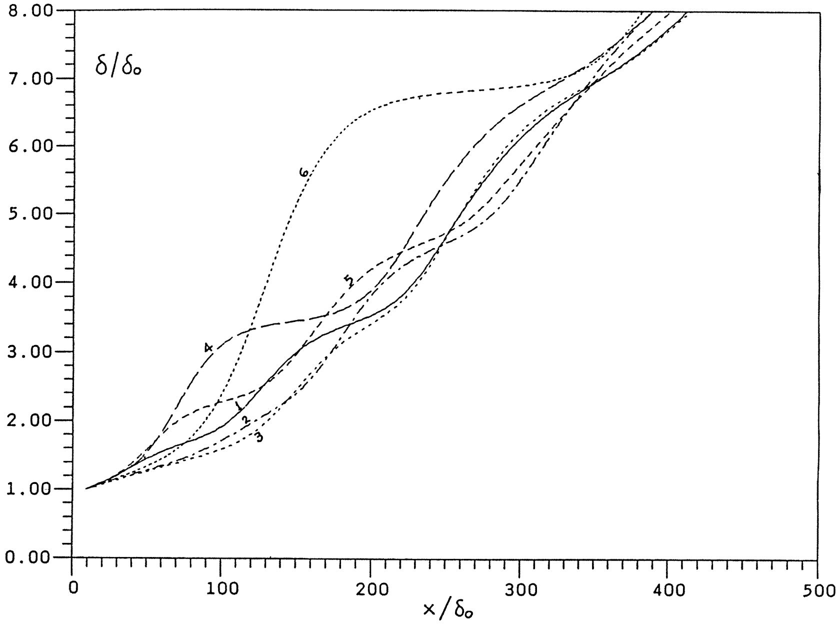

Figure 1. Shear layer growth for different sets of frequency forcing

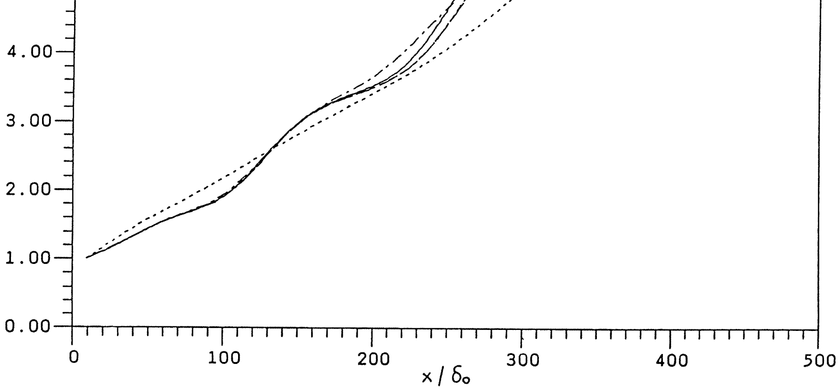

Figure 2. Shear layer growth for different fine-grained turbulence energy levels

In Figure 2, the consequences of varying initial turbulence energy levels in the shear layer are shown: case 1 "standard" $E_0 = 10^{-6}$. For much lower turbulence levels, case 2: $E_0 = 10^{-8}$, case 3: $E_0 = 10^{-12}$ and higher levels, case 4: $E_0 = 10^{-4}$, case 5: $E_0 = 10^{-2}$. The latter correspond to a fully initial turbulent shear layer and much higher coherent mode forcing would be needed than presented here to affect its behavior. The higher turbulence level does not necessarily given a stronger spreading rate downstream whereas the coherent modes render themselves more effective in the spreading rate provided that their energy levels are high. That is, if they are able to extract more energy from the mean flow to overcome the energy transfer to the turbulence. The development of coherent mode amplitudes and turbulence energy will be reported in detail elsewhere.

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