

Contents

Local Organizing Committee	viii
Acknowledgment of Support	viii
Congress Services	viii
0 Plenary Talks	1
Grigor A. Barsegian	1
Alain Bourgeat	1
Paul L. Butzer	2
W. Desmond Evans	2
Bernhard Gramsch	3
Johannes Sjöstrand	4
Cem Yalçın Yıldırım	4
1 Analytic Function Spaces and Their Operators	5
Ramazan Akgün	5
Rauno Aulaskari	5
M. Amelia Bastos	6
Hamid Bounit	6
Cüneyt Çevik	6
Piotr V. Dovbush	6
Paul M. Gauthier	7
Vagif S. Guliyev	7
Uğur Gül	7
Ali Güven	8
Christopher Hammond	8
H. Turgay Kaptanoğlu	8
H. Turgay Kaptanoğlu	8
Yamina Laskri	9
Jasbir S. Manhas	9
Madani Moussai	9
Serap Öztop	9
Lourdes Palacios	10
Qiulan Qi	10
Lino E. Reséndis O.	10
Amol Sasane	11
Luis Manuel Tovar	11
N. Pınar Tozman	11
Frank Wikström	12
Wen Xu	12
Nina Zorboska	12
2 Clifford and Quaternion Analysis	13
Paula Cerejeiras	13
Henni De Schepper	13
Sirkka-Liisa Eriksson	14
Graziano Gentili	14
Uwe Kähler	14
Yakov Krasnov	14
Lukáš Krump	15

Guy Laville	15
Marco A. Macías-Cedeño	15
Tania Moreno-García	16
Dixan Peña-Peña	16
Alessandro Perotti	16
Tao Qian	17
Yuying Qiao	17
Irene M. Sabadini	17
Dalibor Šmid	17
Frank Sommen	18
Le Hung Son	18
Luis Manuel Tovar	18
Fabio Vlacci	19
Zhongxiang Zhang	19
3 Complex Analysis and Potential Theory	20
Polina Agranovich	20
Teodor Bulboacă	20
Serap Bulut	21
Matteo Dalla Riva	21
Dorin Ghisa	21
Anatoly Golberg	22
Alexander Goncharov	22
Daniyal M. Israfilov	22
Olena Karupu	22
Boris A. Kats	23
Vakhtang Kokilashvili	23
Massimo Lanza de Cristoforis	24
Marco A. Macías-Cedeño	24
Jamal I. Mamedkhanov	24
Burçin Oktay	25
Sergiy Plaksa	25
Promarz M. Tamrazov	25
Nikola Tuneski	26
Julia V. Vasilieva	26
Yuri B. Zelinskii	26
4 Complex Analytic Methods for Applied Sciences	27
Anders Andersson	27
Piotr Drygaś	27
Mehdi Eskandarzade	27
Mehdi Eskandarzade	28
Igor M. Gaissinski	28
Misir B. Ragimov	28
Sergei V. Rogosin	29
Ana Moura Santos	29
Tatyana S. Vaitekhovich	30
5 Complex and Functional Analytic Methods in PDEs	31
Ümit Aksoy	31
Mustafa Avcı	31
Heinrich Begehr	32
Natalia Chinchaladze	32

Antonio Di Teodoro	32
Larissa V. Fardigola	33
Grigory Giorgadze	33
George Jaiani	33
Erkinjon Karimov	34
Petra Kordulová	34
Sara Maad	34
Nino Manjavidze	35
Alip Mohammed	35
Nusrat Rajabov	35
Lutfya Rajabova	36
Selmi Ridha	36
Romen S. Saks	37
Zafar D. Usmanov	37
Carmen J. Vanegas	37
Yufeng Wang	37
Yongzhi Xu	38
Uğur Yüksel	38
Oleg Zhdanov	38
Shouguo Zhong	39
6 Dispersive Equations	40
Abbes Benaissa	40
Q-Heung Choi	40
Daniele Del Santo	41
Massimo Fonte	41
Vladimir Georgiev	41
Fumihiko Hirosawa	42
Key Jachmann	42
Tacksun Jung	42
Varga Kalantarov	43
Tamotu Kinoshita	43
Paolo Maremonti	43
Tokio Matsuyama	44
Kiyoshi Mochizuki	44
Hideo Nakazawa	45
Kenji Nishihara	45
Necat Polat	45
Michael Reissig	46
Michael Ruzhansky	46
Hiroshi Uesaka	46
Jens Wirth	47
Borislav Yordanov	47
8 Inverse and Ill-Posed Problems	48
Kamil R. Aida-zade	48
Abdelhani Boukrouche	48
Nadjib Boussetila	49
Fabrizio Colombo	49
Ali Demir	50
Arzu Erdem	50
Samir Z. Guliyev	51

Ruslan A. Hamidov	51
Alexander V. Handzel	52
Alemdar Hasanov	52
Sayed M. Hashemiparast	52
Sergey I. Kabanikhin	53
Tynysbek S. Kalmenov	53
Balgaisha Mukanova	53
Boris Paneah	54
Anar B. Rahimov	54
Zaure Rakisheva	54
Faouzia Rebbani	55
Boris Resnick	55
Firdousi A. Sharifov	56
Yagub A. Sharifov	56
Eva Sincich	57
Salih Tatar	57
Gerhard-Wilhelm Weber	58
Yongzhi Xu	58
Karlyga Zhilisbaeva	58
10 Modern Aspects of Theory of Integral Transforms and Their Applications	59
Aigul Aktymbayeva	59
Dong Hyun Cho	59
Biljana Jolevska-Tuneska	60
Anatoly A. Kilbas	60
Anatoly A. Kilbas	61
Bong Jin Kim	61
Byoung Soo Kim	61
Anna A. Koroleva	62
Erlan D. Nursultanov	62
Zhaxylyk N. Tasmambetov	63
Il Yoo	63
11 Oscillation of Functional-Differential and Difference Equations	64
M. Fahri Aktaş	64
Yaşar Bolat	64
Josef Diblík	64
Josef Diblík	65
Gusein S. Guseinov	65
Billur Kaymakçalan	65
Roman Koplatadze	65
Raziye Mert	66
Abdullah Özbekler	66
Boris Paneah	66
Fatoş Tuncay	67
Mehmet Ünal	67
Ahmet Yantır	67
Ağacık Zafer	67
12 Pseudodifferential Operators	68
Viorel Catană	68
Rachid Chaili	68
Ahmed Dali	69

Aparajita Dasgupta	69
Gianluca Garelo	69
Claudia Garetto	69
Juan B. Gil	70
Todor Gramchev	70
Chisato Iwasaki	70
Jon Johnsen	70
Thomas Krainer	71
Yu Liu	71
Alip Mohammed	71
Michael Oberguggenberger	72
Stevan Pilipović	72
Frederic Rochon	72
Michael Ruzhansky	73
Bert-Wolfgang Schulze	73
Joachim Toft	73
Ville Turunen	74
Tomio Umeda	74
Seiichiro Wakabayashi	74
M. W. Wong	75
13 Reproducing Kernels and Related Topics	76
Keiko Fujita	76
Tsutomu Matsuura	76
Renata Otáhalová	77
Saburou Saitoh	77
14 Spaces of Differentiable Functions of Several Real Variables and Applications	78
Sadig K. Abdullayev	78
Alexandre Almeida	78
Tsegaye G. Ayele	78
Bakhyt Baideldinov	79
Viktor I. Burenkov	79
Douadi Drihem	79
İsmail Ekincioglu	80
Amiran Gogatishvili	80
Alexander Goncharov	80
Vagif S. Guliyev	81
Javanshir J. Hasanov	81
Alakber A. Huseynov	82
Vakhtang Kokilashvili	82
Leili Kussainova	82
Pier Domenico Lamberti	83
Alexander Meskhi	83
Necip Özfidan	84
Vakhtang Paataashvili	84
Humberto Rafeiro	84
Zaman V. Safarov	84
Stefan G. Samko	85
I. I. Sharapudinov	85
Tamara Tararykova	86
Askhab Y. Yakubov	86

15 Numerical Functional Analysis	87
Allaberen Ashyralyev	87
Abdumauvlen S. Berdyshev	87
Valéry Covachev	88
Abdullah Said Erdoğan	88
Okan Gerçek	88
Esat Gezgin	89
Mehmet Emir Köksal	89
Alexey L. Lukashov	89
Minoru Nakano	89
David Natroshvili	90
David Natroshvili	90
Yıldırım Özdemir	91
Elif Öztürk	91
Javier Pastor	92
Yaşar Sözen	92
16 Integrable Systems	93
Barış Ateş	93
Valentina A. Golubeva	93
Ismagil Habibullin	94
Satyendra N. Pandey	94
Ash Pekcan	94
Anatoliy K. Prykarpatsky	95
Burcu Silindir	95
Kostyantyn Zheltukhin	95
17 General Session	96
Dahmane Achour	96
Fathi M. Allan	96
Mersaid Aripov	96
Ülkü Dinlemez Ateş	97
Zbigniew Bartosiewicz	97
Benyattou Benabderahmanne	97
Rabin K. Bhattacharyya	98
Ilya V. Boykov	98
Pallath Chandran	99
Mourad Dilmi	99
Valery A. Gaiko	99
Igor M. Gaissinski	100
V. I. Gishlarkaev	100
Mohamed Ali Hajji	100
Baltabek E. Kanguzhin	101
Erdal Karapınar	101
Bülent Karasözen	101
Meryem Kaya	102
Shaghayegh Kordnourie	102
Djeddi Mabrouk	102
H. P. Masiha	103
Hamza Menken	103
Lahcène Mezrag	103
Oleksii Mokhonko	104

Kourosch Nourouzi	104
Mukhtarbay Otelbaev	105
Ali Özyapıcı	105
Shahram Rezapour	106
Khalil Saadi	106
Nirmal C. Sacheti	106
Katerina Saneva	107
Kamal N. Soltanov	107
Celalettin Şençimen	107
Tanfer Tanrıverdi	108
Ahmadjon K. Urinov	108
Index	109

Local Organizing Committee

Öner Çakar (*Ankara University, Turkey*)

A. Okay Çelebi (*Middle East Technical University, Ankara, Turkey*)

H. Turgay Kaptanoğlu (*Bilkent University, Ankara, Turkey*)

Ağacık Zafer (*Middle East Technical University, Ankara, Turkey*)

Acknowledgment of Support

Republic of Turkey Prime Ministry Promotion Fund

Turkish International Cooperation and Development Agency (TİKA)

Scientific and Technological Research Council of Turkey (TÜBİTAK)

Middle East Technical University (ODTÜ)

Ankara University

Bilkent University

Gama Trade and Tourism

Yüksel Construction

Kavaklıdere Anatolian Wines

Ertem Printing House

Birkhäuser

And volunteer work of colleagues and friends too many to mention here.

Congress Services

Asterya Congress & Event Management

0 Plenary Talks

Some new inequalities in geometry and analysis

GRIGOR A. BARSEGHIAN

Academy of Sciences, Yerevan, Armenia

barseg@instmath.sci.am

In this talk we present some new inequalities that refer to broken lines, curves, real and complex functions. Their derivation is based on a new *principle of angles and lengths* for curves. The inequality in the complex analysis, called the *principle of derivatives*, is valid for analytic functions in arbitrary domains and extends to a broad class of sufficiently smooth complex functions. A new inequality follows for real functions of two variables concerning their level sets. For all the mentioned above cases, curves and functions, we obtain some analogues of the second fundamental theorem in Nevanlinna theory of meromorphic functions. At the end we discuss a new *point-domain* inequality dealing with finite point sets in an arbitrary domain.

Spatial and random scaling up of the source terms in a diffusion convection model

ALAIN BOURGEAT

Université de Lyon, France

bourgeat@mcs.univ-lyon1.fr

We study the transport migration of contaminants in an aquifer from a “sources site” made of a large number of “local” sources, assuming uncertainty on the behavior of each local source. The local sources f^ε are periodically repeated and lying on a plan; and their release curve parameters are considered random in space and in time. For this, first, we define ε as a small positive number (measuring the typical length of a source support), and assume that each source has its support K_ε in a thin parallelepipedic set:

$$K_\varepsilon = \varepsilon([0, s_1] \times [0, s_2] \times \varepsilon^{\gamma-1}[-s_3, s_3]).$$

Assuming that the sources density f^ε is statistically homogeneous with respect to the 2D variable $x' = (x_1, x_2)$, we write the “sources term”

$$f^\varepsilon(x, t) = \mathbb{I}_{B_\varepsilon} \frac{1}{\varepsilon^\gamma} \phi(T_{\mathbf{z}}\omega, t)$$

with $\{T_{\mathbf{z}} : \mathbf{z} \in \mathbb{Z}^2\}$ a discrete random ergodic dynamical system. Then the initial-boundary problem describing the spatial evolution of the concentration u^ε reads:

$$\begin{aligned} \partial_t u^\varepsilon - \operatorname{div}(a(x)\nabla u^\varepsilon) + \operatorname{div}(b(x)u^\varepsilon) &= f^\varepsilon, \quad \text{in } Q \times (0, \infty); \\ u^\varepsilon|_{t=0} &= 0, \quad \frac{\partial}{\partial n_a} u^\varepsilon - b(x) \cdot n(x)u^\varepsilon + \lambda u^\varepsilon = 0 \quad \text{on } \partial Q \times (0, \infty), \end{aligned}$$

We derive, by homogenization, a global model describing the spatial evolution of a global concentration u^0 , which has now only one deterministic “global” source term on the right hand side. Adding different mixing properties to the previous general assumptions, we estimate the rate of convergence expectation for $(u^\varepsilon - u^0)$; and we may prove the convergence in distribution for the corrector $u^1 = (u^\varepsilon - u^0)/\varepsilon$ at any fixed point (x, t) to a centered Gaussian random variable. Finally, assuming that the time trajectory $t \mapsto f^\varepsilon(x, t)$ is not random, we obtain the same convergence in distribution for any finite joint distribution.

In conclusion, in order to illustrate our results, we use the above model for describing a long-lived nuclear waste underground repository, and we present numerical simulations showing the accuracy of our theoretical results obtained herein by homogenization.

Prediction in terms of samples from the past: Error estimates by a modulus covering discontinuous signals

PAUL L. BUTZER

Rheinisch-Westfälische Technische Hochschule, Aachen, Germany

butzer@rwth-aachen.de

Coauthor: R. L. Stens

This paper is concerned with the prediction or extrapolation of signals in terms of past samples. It is carried out by means of discrete convolution sums the kernels of which have compact support so that only a finite number of samples from the past is needed. The signal functions, assumed to be duration limited, need not be continuous, even though sharp error estimates in the $L^p(\mathbb{R})$ -norm are incorporated.

The convolution sums considered for f in a certain subspace Λ^p of $L^p(\mathbb{R})$, $1 \leq p < \infty$, are

$$(S_W^\varphi f)(t) := \sum_{k=-\infty}^{\infty} f\left(\frac{k}{W}\right) \varphi(Wt - k) \quad (W > 0),$$

the kernels $\varphi \in L^1(\mathbb{R})$ having support in $[T_0, T_1] \subset (0, \infty)$, so that φ vanishes at most for those $k \in \mathbb{Z}$ for which $k/W \in (t - T_1/W, t - T_0/W]$. Thus to predict $f(t)$ (the $L^p(\mathbb{R})$ -limit of $S_W^\varphi f$ for $W \rightarrow \infty$) at time t only a finite number of samples from the past, strictly less than t , are needed. The kernels studied in detail include linear combination of B-splines such as $\varphi_1(t) := 3M_2(t-2) - 2M_2(t-2)$, $\varphi_2(t) := 1/8\{47M_3(t-2) - 62M_3(t-3) + 23M_3(t-4)\}$, and discontinuous signals as $f_1(t) = -4/9t^2 + 2$ for $-7/4 < t < 1$, and equal $|t|^{-3}$, elsewhere, which has jump discontinuities at $t = -7/4$ and $t = 1$.

It is possible to predict arbitrarily far ahead, at the cost, however, of a rather large error. Also prediction of derivatives of signals in terms of samples of the the signals themselves is investigated. Best possible error estimates for (discontinuous) signals cannot be given in terms of the classical modulus of continuity $\omega_r(f; \delta; L^p(\mathbb{R}))$ but in terms of the $L^p(\mathbb{R})$ -averaged modulus of smoothness (or τ -modulus), investigated recently by Bardaro, Butzer, Stens & Vinti [*J. Math. Anal. Appl.* **316** (2006), 269–306] in the case of the classical Whittaker-Kotel'nikov-Shannon sampling theorem in the $L^p(\mathbb{R})$ -norm for $1 < p < \infty$.

Recent results on the Hardy and Rellich inequalities

W. DESMOND EVANS

Cardiff University, Wales, UK

evanswd@cardiff.ac.uk

The Hardy inequality is well-known to have an important role in many branches of analysis and mathematical physics, and continues to be the subject of intensive study and research. The lecture will survey recent results on the inequality, both in \mathbb{R}^n and also on bounded domains, involving the distance to the boundary of the domain, and discuss some important implications. Analogous results for the Rellich inequality will also be given.

Spectral invariance for pseudodifferential operators and Fredholm theory

BERNHARD GRAMSCH

Gutenberg-Universität, Mainz, Germany

gramsch@imamz74.mathematik.uni-mainz.de

For the analytic and the topological properties of holomorphic Fredholm functions in Fréchet algebras \mathcal{A} of Fourier operators it is useful to point out the following crucial properties:

1. The Fréchet algebra \mathcal{A} is continuously embedded in a Banach algebra \mathcal{B} with unit e (e.g. $\mathcal{B} = \mathcal{L}(\mathcal{E})$, \mathcal{E} a Banach space) such that there exists an $\varepsilon > 0$ with

$$\{a \in \mathcal{A} : \|a - e\| < \varepsilon\} \subseteq \mathcal{A}^{-1} \quad (\text{I})$$

(local spectral invariance of \mathcal{A} in \mathcal{B}), where \mathcal{A}^{-1} denotes the group of invertible elements of \mathcal{A} .

2. \mathcal{A} is submultiplicative: the topology on \mathcal{A} is given by a countable system of norms $\{p_k : k = 0, 1, \dots\}$ such that for all $a, b \in \mathcal{A}$ and for all k

$$p_k(ab) \leq p_k(a)p_k(b). \quad (\text{II})$$

The spectral invariance of \mathcal{A} in \mathcal{B} (\mathcal{A} is inverse closed in \mathcal{B})

$$\mathcal{A} \cap \mathcal{B}^{-1} = \mathcal{B}^{-1} \quad (\text{III})$$

follows from (I) if \mathcal{A} is in addition a symmetric subalgebra of a C^* -algebra \mathcal{B} (e.g. $\mathcal{B} = \mathcal{L}(\mathcal{H})$, \mathcal{H} a Hilbert space).

Remark. The Hörmander classes $\Psi_{\varrho, \delta}^0$ ($0 \leq \delta \leq \varrho \leq 1$, $\delta < 1$), some Boutet de Monvel classes of boundary value problems and the calculus of Melrose and Schulze is related to (I), (II) and (III) by results of e.g. Lauter, Nistor and Schrohe.

Remark. The properties (I) and (II) are stable under taking closed subalgebras of \mathcal{A} or countable intersections. This leads to a new functional analytic construction of generalized pseudodifferential operators on manifolds with singularities, on nets and on ramified spaces. Elliptic regularity and the propagation of singularities can be discussed in this setting.

It turns out that in the set of left semi-Fredholm operators

$$\Phi^l := \{a \in \mathcal{A} : \exists b \in \mathcal{A} \text{ with } aba = a \text{ and } \dim \ker a < \infty\},$$

the set $\Phi_k^l := \{a \in \Phi^l : \dim \ker a = k\}$ is a direct analytic submanifold of \mathcal{A} and a homogeneous space with rational structure.

Theorem. (α) Let Ω be a holomorphy region and $T : \Omega \rightarrow \Phi^l$ a holomorphic map such that for some $z^* \in \Omega$ there exists a left inverse for $T(z^*)$ in \mathcal{A} (\mathcal{A} with (I)). Then there exists a meromorphic left inverse $M(z)$ of $T(z)$ on Ω with a decomposition $M(z) = a(z) + s(z)$, where $a : \Omega \rightarrow \mathcal{A}$ is holomorphic and $s(z)$ is meromorphic with values in the ideal of operators with fast decreasing approximation numbers.

(β) Let Ω be as above. $\mathcal{H}(\Omega, M)$ resp. $\mathcal{C}(\Omega, M)$ denotes the space of holomorphic resp. continuous maps from Ω into the Fréchet manifold $M := \Phi_k^l$. Then we have the bijection

$$\pi_n(\mathcal{H}(\Omega, M)) \cong \pi_n(\mathcal{C}(\Omega, M)) \quad (n = 0, 1, \dots)$$

for all homotopy sets (Oka-Grauert principle).

We discuss recent extensions, applications and limitations of (α) and (β).

The dissertations (at Mainz) of W. Bauer (2005), M. Höber (2006) and J. Ditsche (2007) provide essential contributions to Fréchet operator algebras with (I) and (II) above.

Non-self-adjoint spectral problems and the Kramers-Fokker-Planck operator

JOHANNES SJÖSTRAND

Ecole Polytechnique, Palaiseau, France

johannes@math.polytechnique.fr

Coauthors: M. Hitrik, F. Hérau, S. Vũ Ngọc, C. Stolk, et al.

Non-self-adjoint operators present many interesting phenomena and difficulties. In this talk, we first discuss the pseudospectral phenomenon, that is the possibility that the norm of the resolvent may be large even when the spectral parameter is far from the spectrum.

Then we discuss some recent results for analytic differential operators in dimension 2, where the pseudospectral difficulty often can be overcome by modifying the ambient Hilbert space by means of microlocal exponential weights, leading to a complete asymptotic description of the eigenvalues.

After that, we turn to the Kramers-Fokker-Planck equation and discuss the complete asymptotics of low-lying eigenvalues in the semi-classical (low temperature) limit, including tunnel effects and return to equilibrium. Here bounded exponential weights on phase space are used.

Recent results on small gaps between primes

CEM YALÇIN YILDIRIM

Boğaziçi University & Feza Gürsey Institute, İstanbul, Turkey

yalciny@boun.edu.tr

In recent works by D. A. Goldston, J. Pintz and C. Y. Yıldırım, the use of short divisor sums has led to quite strong results concerning the existence of small gaps between primes.

The results depend on the information about the distribution of primes in arithmetic progressions, specifically on the range where the Bombieri-Vinogradov Theorem is taken to hold. It is shown unconditionally that

$$\liminf_{n \rightarrow \infty} \frac{p_{n+1} - p_n}{\log p_n} = 0,$$

along with the quantitative version

$$\liminf_{n \rightarrow \infty} \frac{p_{n+1} - p_n}{\sqrt{\log p_n} (\log \log p_n)^2} < \infty$$

(here p_n denotes the n -th prime). Assuming that the Bombieri-Vinogradov Theorem holds with any level beyond the known level $\frac{1}{2}$, the method establishes the existence of bounded gaps between consecutive primes. Some further results concerning the differences $p_{n+\nu} - p_n$ (ν : any fixed positive integer) are also obtained. The corresponding problem for E_2 -numbers q_n , numbers which are the product of two distinct primes, have been studied by S. Graham and the three mentioned researchers. In this case it is proved that

$$\liminf_{n \rightarrow \infty} (q_{n+1} - q_n) \leq 6.$$

In my talk I shall try to give a presentation of the main ideas involved in these works.

1 Analytic Function Spaces and Their Operators

Organizers

Rauno Aulaskari (*University of Joensuu, Finland*)

H. Turgay Kaptanoğlu (*Bilkent University, Ankara, Turkey*)

Approximation by rational functions in Smirnov-Orlicz classes

RAMAZAN AKGÜN

Balikesir University, Turkey

rakgun@balikesir.edu.tr

Coauthor: D. M. Israfilov

Let $\Gamma \subset \mathbb{C}$ be a Dini-smooth curve and let G^- be the unbounded component of Γ . We suppose that a weight function ω defined on Γ satisfies the Muckenhoupt's $A_{1/\alpha_M}(\Gamma) \cap A_{1/\beta_M}(\Gamma)$ conditions where α_M, β_M are nontrivial Boyd indices of weighted Smirnov-Orlicz class $E_M(G^-, \omega)$. Let $L_M(\mathbb{T}, \omega)$ be the weighted Orlicz space defined on the unit circle \mathbb{T} . We define the moduli of smoothness of order $r = 1, 2, \dots$ of a function $f \in E_M(G^-, \omega)$ as

$$\Omega_{\Gamma, M, \omega}^r(f, \delta) := \sup_{\substack{0 < h_i \leq \delta \\ i=1, \dots, r}} \left\| \prod_{i=1}^r (I - \sigma_{h_i}) f(e^{i\theta}) \right\|_{L_M(\mathbb{T}, \omega \circ \psi)} \quad (\delta > 0),$$

where $(\sigma_h f)(w) := \frac{1}{2h} \int_{-h}^h f(we^{it}) dt$, $w \in \mathbb{T}$, $0 < h < \pi$, and ψ is the normalized conformal mapping of the unbounded component of \mathbb{T} onto the bounded component of Γ . For $f \in E_M(G^-, \omega)$, we set $E_n(f)_{M, \omega} := \inf_{R \in \mathcal{R}_n} \|f - R\|_{E_M(G^-, \omega)}$, where \mathcal{R}_n is the set of rational functions of the form $\sum_{k=0}^n a_k z^{-k}$. We prove direct and converse theorems of rational approximation in $E_M(G^-, \omega)$ classes. A constructive characterization of generalized Lipschitz classes is obtained.

Theorem 1. Let $f \in E_M(G^-, \omega)$. Then $\|f - R_n(\cdot, f)\|_{E_M(G^-, \omega)} \leq c \Omega_{\Gamma, M, \omega}^r(f, 1/(n+1))$ for $r = 1, 2, 3, \dots$, where $R_n(\cdot, f)$ is the n -th partial sum of the Faber-Laurent series of f .

Theorem 2. Let $f \in E_M(G^-, \omega)$. Then

$$\Omega_{\Gamma, M, \omega}^r(f, 1/n) \leq \frac{c}{n^{2r}} \left\{ E_0(f)_{M, \omega} + \sum_{k=1}^n k^{2r-1} E_k(f)_{M, \omega} \right\}, \quad r = 1, 2, 3, \dots$$

On the growth of Dirichlet integral for some function spaces

RAUNO AULASKARI

University of Joensuu, Finland

rauno.aulaskari@joensuu.fi

Coauthor: S. Makhmutov

The growth of Dirichlet integral for functions belonging to Q_p but not to the Dirichlet space is considered. These results are shown to be strict in a sense.

An invertibility criterion for C^* -algebras of functional operators with shifts

M. AMELIA BASTOS

Instituto Superior Tecnico, Lisbon, Portugal

abastos@math.ist.utl.pt

Coauthors: C. Fernandes & Y. Karlovich

A non-local C^* -algebras, generated by multiplication operators by slowly oscillating and piecewise continuous functions and by the range of a unitary representation of an amenable group of diffeomorphisms with any non-empty set of common fixed points, is investigated. A generalization of the local trajectory method for C^* -algebras associated with C^* -dynamical systems, based on the notion of spectral measures, allows to establish an invertibility criterion for the elements of this C^* -algebra.

A note on the Weiss conjecture for analytic semigroups

HAMID BOUNIT

Université Ibn Zohr, Agadir, Morocco

bounith@yahoo.fr

Coauthor: O. El-Mennaoui

Let $\mathbb{T}(t)$ be an analytic semigroup with generator A on some Banach space. Le Merdy [*J. London Math. Soc.* **67** (2003), 715–738] showed the validity of the so-called Weiss conjecture: $C \in \mathcal{L}(D(A), Y)$ is an admissible observation operator if and only if $\{C(\lambda I - A)^{-1} : \operatorname{Re} \lambda \geq \alpha \text{ for some } \alpha > 0\}$ is a bounded set, is equivalent to the finite-time admissibility of the fractional power $(-A)^{1/2}$ for A , where Y is another Banach space. The proof is essentially based on the H^∞ functional calculus. Here we give a new (and a much shorter) proof of this result that does not make any recourse to the H^∞ functional calculus. Next we show that the analyticity assumption of $\mathbb{T}(t)$ cannot be omitted. Le Merdy gave sufficient conditions regarding A under which the fractional power $(-A)^{1/2}$ is an admissible observation operator for A . In this note it is shown that this may happen for analytic and normal semigroups on Hilbert space. This constitutes a new proof of the Weiss conjecture for normal and analytic semigroups.

Some properties of projective tensor product of $L^1(\mu)$ and a Banach lattice

CÜNEYT ÇEVİK

Gazi University, Ankara, Turkey

ccevik@gazi.edu.tr

Let E be a Banach lattice and $L^1(\mu, E)$ be the space of E -valued Bochner integrable functions. We illustrate that some properties of $L^1(\mu, E)$ are inherited from $L^1(\mu)$ and E , and some properties of E and $L^1(\mu)$ are consequences of properties of $L^1(\mu, E)$.

On Bloch functions and normal functions on complex Banach manifolds

PIOTR V. DOVBUSH

Academy of Sciences, Chişinău, Moldova

peter.dovbush@gmail.com

Following Lehto and Virtanen 1957 and Pommerenke 1970, we generalize the notions of normal function, Bloch function and other notions of one complex variable to the holomorphic functions on a complex Banach manifold modelled on a complex Banach space of positive, possibly infinite, dimension and obtain various relations which exist among them. Several necessary and sufficient conditions for holomorphic functions to be normal/Bloch are established. Boundary behavior of normal functions defined on a bounded domain in a complex Banach space, are also studied. We formulate and prove a Lindelöf principle for admissible approach regions.

Approximation by bounded holomorphic functions

PAUL M. GAUTHIER

Université de Montréal, Canada

gauthier@dms.umontreal.ca

For G an open set of the Riemann sphere, let $H(G)$ and $B(G)$ denote the families of holomorphic and bounded holomorphic functions respectively on G , and let $A(G)$ denote the family of holomorphic functions on G which extend continuously to the closure of G . Raymond Mortini asked for a characterization of those domains G for which $B(G)$ is dense in $H(G)$. Jointly with Mark Melnikov, we give such a characterization as well as a characterization of those G for which $A(G)$ is dense in $H(G)$.

Strong type estimates and Carleson measures for weighted Besov spaces

VAGIF S. GULIYEV

Academy of Sciences, Baku, Azerbaijan

vagif@guliyev.com

Coauthor: Z. Wu

We establish a capacity strong type estimate for weighted Besov spaces $B_{pq}^{\alpha,\varphi}(\mathbb{R}^n)$ and characterize the related Carleson measures. For $p, q \geq 1$ and $0 < \alpha < 1$, the weighted Besov-Lipschitz space $B_{pq}^{\alpha,\varphi}(\mathbb{R}^n)$ consists of all functions f in $L_{p,\varphi} = L_p(\mathbb{R}^n, \varphi(x)dx)$ so that

$$\|f\|_{B_{pq}^{\alpha,\varphi}} = \|f\|_{p,\varphi} + \left(\int_{\mathbb{R}^n} \frac{\|f(\cdot + h) - f(\cdot)\|_{p,\varphi}^q}{|h|^{n+\alpha q}} dh \right).$$

For any open set $\mathcal{O} \subset \mathbb{R}^n$, $\text{cap}_{B_{pq}^{\alpha,\varphi}}(\mathcal{O}) = \inf \{ \|f\|_{B_{pq}^{\alpha,\varphi}(\mathbb{R}^n)} : f \geq 1 \text{ on } \mathcal{O} \}$ defines the $B_{pq}^{\alpha,\varphi}(\mathbb{R}^n)$ -capacity of \mathcal{O} . Let $\mathbb{R}_+^{n+1} = \{(x, t) : x \in \mathbb{R}^n, t > 0\}$ be the upper half space. For any open set $\mathcal{O} \subset \mathbb{R}^n$, the tent of \mathcal{O} in \mathbb{R}_+^{n+1} is $T(\mathcal{O}) = \{(x, t) \in \mathbb{R}_+^{n+1} : \text{open ball with center } x \text{ and radius } t \text{ is contained in } \mathcal{O}\}$. Let $P_t(x) = c_n t (|x|^2 + t^2)^{-\frac{n+1}{2}}$ be the normalized Poisson kernel. For any function $f \in L_{p,\varphi}$, the harmonic extension of f to \mathbb{R}_+^{n+1} is the convolution $F(x, t) = P_t * f(x)$ between P_t and f .

Theorem 1. For $1 \leq p \leq q \leq \infty$ and a weight φ , there is a $C > 0$ such that the strong type estimate $\int_0^\infty \text{cap}_{B_{pq}^{\alpha,\varphi}}(|f| > t) dt^q \leq C \|f\|_{B_{pq}^{\alpha,\varphi}}^q$ holds for all $f \in B_{pq}^{\alpha,\varphi}$.

Theorem 2. Suppose $1 < p \leq q \leq \infty$ and φ is a weight. A nonnegative measure μ on \mathbb{R}_+^{n+1} satisfies $\int_{\mathbb{R}_+^{n+1}} |P_t * f(x)|^q d\mu(x, t) \leq C \|f\|_{B_{pq}^{\alpha,\varphi}}^q$ for all $f \in B_{pq}^{\alpha,\varphi}$ if and only if $\mu(T(\mathcal{O})) \leq C \text{cap}(\mathcal{O}; B_{pq}^{\alpha,\varphi})$ for any bounded open set $\mathcal{O} \subset \mathbb{R}^n$.

Essential spectra of composition operators on Hardy spaces

UĞUR GÜL

Middle East Technical University, Ankara, Turkey

gulugur@gmail.com

We study essential spectra of composition operators on the Hardy space of the unit disc and of the upper half-plane. Our starting point is the spectral analysis of the composition operators induced by translations of the upper half-plane. We completely characterize the essential spectra of a class of composition operators that are induced by perturbations of translations.

Two-weight estimates for Fourier operators and Bernstein inequality

ALİ GÜVEN

Balıkesir University, Turkey

ag_guven@yahoo.com

Coauthor: V. Kokilashvili

The norm estimation problem for Fourier operators acting from $L_w^p(\mathbb{T})$ to $L_v^q(\mathbb{T})$ where $1 < p \leq q < \infty$ was investigated. These results were generalized to the two-dimensional case and applied to obtain generalizations of the Bernstein inequality for trigonometric polynomials of one and two variables. Also, the rates of convergence of Cesaro and Abel-Poisson means of functions $f \in L_w^p(\mathbb{T})$ were estimated in the case $p = q$ and $v = w$. The generalized Bernstein inequality applied to estimate the order of best trigonometric approximation of the derivative of functions $f \in L_w^p(\mathbb{T})$ in the space $L_v^q(\mathbb{T})$.

Adjoint of composition operators with rational symbols

CHRISTOPHER HAMMOND

Connecticut College, New London, CT, USA

cnham@conncoll.edu

Coauthors: J. Moorhouse & M. E. Robbins

Building on techniques developed by Cowen and Gallardo-Gutiérrez, we find a concrete formula for the adjoint of a composition operator with rational symbol acting on the Hardy space H^2 . We consider some specific examples, comparing our formula with several results that were previously known.

Toeplitz C^* -algebras on Dirichlet spaces of the ball

H. TURGAY KAPTANOĞLU

Bilkent University, Ankara, Turkey

kaptan@fen.bilkent.edu.tr

Dirichlet spaces \mathcal{D}_q ($q \in \mathbb{R}$) are Hilbert spaces of holomorphic functions on the unit ball of \mathbb{C}^N defined by a reproducing kernel which is $K_q(z, w) = (1 - \langle z, w \rangle)^{-(N+1+q)}$ for $q > -(N+1)$ and given by a hypergeometric function for $q \leq -(N+1)$.

We consider the C^* -algebra \mathcal{T}_q generated by the N -tuple of operators of multiplication by the coordinate functions (the so-called N -shift) on \mathcal{D}_q . We show that \mathcal{T}_q contains all compact operators and Toeplitz operators with continuous symbols, a quotient map sends it onto the continuous functions on the boundary of the unit ball, and thus obtain the related short exact sequence of C^* -algebras. This result generalizes what is known for the Hardy space ($q = -1$), Bergman spaces ($q > -1$), and the Drury-Arveson space ($q = -N$).

We also show that the symmetric Fock space over \mathbb{C}^N can be realized as each of the Dirichlet spaces under a suitable norm as noticed earlier for the Arveson space ($q = -N$).

Bergman projections on weighted Bloch and Lipschitz spaces

H. TURGAY KAPTANOĞLU

Bilkent University, Ankara, Turkey

kaptan@fen.bilkent.edu.tr

Coauthor: S. Tülü

Precise conditions are determined for the boundedness of Bergman projections from Lebesgue classes onto the spaces in the title. The projections provide integral representations for the functions in the spaces. Many properties of the spaces are obtained as straightforward corollaries of the projections and the integral representations.

Strong asymptotic formula for L^p extremal polynomials on the circle

YAMINA LASKRI

University of Annaba, Algeria

minalaskri@yahoo.fr

Coauthors: M. Belhout & R. Benzine

For all $p > 0$, we establish a strong asymptotic formula for L^p extremal polynomials corresponding to a measure with infinite discrete part off the unit circle

Topological structures of the spaces of composition operators on spaces of analytic functions

JASBIR S. MANHAS

Sultan Qaboos University, Oman

manhas@squ.edu.om

Let $C(X)$ and $C_w(X)$ denote the spaces of composition operators and weighted composition operators on a Banach space X , respectively, equipped with operator norm topologies. The main aim of this talk is to present a survey of topological structures (e.g., component structure, isolated points, compact differences, etc.) of the spaces $C(X)$ and $C_w(X)$ when X is a Hardy space, Bergman space, Dirichlet space, H^∞ -space, Bloch space and weighted Banach space of analytic functions. Some open problems are given for further investigation.

Composition theorems in Besov spaces, the vector-valued case

MADANI MOUSSAI

University of M'Sila, Algeria

mmoussai@yahoo.fr

In the Besov space $B_{p,q}^s(\mathbb{R})$ and under some extra conditions on the parameters s, p, q , and β , we shall study the composition defined by $T_f(g) = f \circ g$, where $g \in B_{p,q}^s(\mathbb{R}, \mathbb{R}^2)$, and f is a real Lipschitz continuous function on \mathbb{R}^2 , vanishing at the origin and the first derivatives belong to $B_{p,q}^{s-1}(\mathbb{R}^2) \cap \text{Lip}_\beta(\mathbb{R}^2)$.

A note on weighted L^1 -modules

SERAP ÖZTOP

Istanbul University, Turkey

oztops@istanbul.edu.tr

Let G be a locally compact abelian group and ω be a weight function on G . We study the $L_\omega^1(G)$ -modules and their multipliers. We also show that there exists a close connection between the almost periodicity of an element of a $L_\omega^1(G)$ -module E and the compactness of an operator from $L_\omega^1(G)$ to E . In the particular case $\omega = 1$, we obtain $L^1(G)$ -modules.

On topologically invertible elements in topological algebras

LOURDES PALACIOS

Universidad Autonoma Metropolitana, Iztapalapa, Mexico

pafa@xanum.uam.mx

Coauthors: H. Arizmendi & A. Carrillo

M. Wojciechowski and W. Zelazko defined a family of topologies τ_c for the algebra $P(t)$ of all complex polynomials and proved that $P(t)$ endowed with any of these topologies is a complete locally convex algebra which is not a Q -algebra. We define the notion of a Q_t -algebra and show that each algebra $(P(t), \tau_c)$ has this property. We characterize the topologically invertible elements in $(P(t), \tau_c)$. In this talk I will show that the boundary $\delta(G_t(P(t)))$ of the set $G_t(P(t))$ of all the topologically invertible elements of $(P(t), \tau_c)$ consists, except for the zero polynomial, only of strong topological divisors of zero.

Approximation properties for certain mixed Szász-beta operators in L^p spaces

QIULAN QI

Hebei Normal University, Shijiazhuang, China

qiqiulan@hebtu.edu.cn

In this paper, we study the mixed summation-integral type operators having Szász and beta basis functions. We obtain the direct, inverse and equivalent theorems in the spaces L_p ($1 \leq p \leq \infty$), using the Ditzian-Totik modulus of smoothness $\omega_\varphi(f, t)$. We obtain the pointwise approximation direct, inverse and equivalence theorems, using the unified modulus of smoothness $\omega_{\varphi^\lambda}^2(f, t)$ ($0 \leq \lambda \leq 1$). A strong converse inequality of type B is given. By this inequality, the converse theorem can be obtained for the operators.

Weighted hyperbolic Bergman classes

LINO E. RESÉNDIS O.

Universidad Autonoma Metropolitana, Azcapotzalco, Mexico

lfro@correo.azc.uam.mx

Let Δ be the open unit disk in the complex plane and $|a| < 1$. Green's function with singularity at a is $g(a, z) = \ln(|1 - \bar{a}z||z - a|^{-1})$. Let $0 \leq s < \infty$ and $1 < p < \infty$. We say that an analytic function $f : \Delta \rightarrow \Delta$ belongs to the hyperbolic Bergman class $\mathcal{A}_{p,s}^*$ if

$$\sup_{a \in \Delta} \iint_{\Delta} \frac{|f(z)|^p}{(1 - |f(z)|^2)^2} g^s(z, a) dx dy < \infty.$$

The theory of \mathcal{Q} spaces was introduced by Aulaskari and Lappan in 1994 and since then has been actively studied. In 2005 Xionan Li, in her Ph.D. thesis, introduced the so-called hyperbolic Q classes. We continue this line of research and present some properties about these classes $\mathcal{A}_{p,s}^*$ and some other related spaces, in particular the euclidean Bergman spaces $\mathcal{A}_{p,s}$.

Properties of a subalgebra of $H^\infty(\mathbb{D})$ and stabilization

AMOL SASANE

London School of Economics, UK

A.J.Sasane@lse.ac.uk

Let \mathbb{D} denote the open unit disk $\{z \in \mathbb{C} \mid |z| < 1\}$ and \mathbb{T} be the unit circle $\{z \in \mathbb{C} \mid |z| = 1\}$. If S is an open subset of \mathbb{T} , then $A_S(\mathbb{D})$ denotes the set of all functions $f : \mathbb{D} \cup S \rightarrow \mathbb{C}$ that are holomorphic in \mathbb{D} and are bounded and continuous in $\mathbb{D} \cup S$. Equipped with the supremum norm, $A_S(\mathbb{D})$ is a Banach algebra, and it lies between the extreme cases of the disk algebra $A(\mathbb{D})$ ($S = \mathbb{T}$) and the Hardy space $H^\infty(\mathbb{D})$ ($S = \emptyset$). We prove the following:

1. The “best norm estimates” of solutions in the corona theorem for $A_S(\mathbb{D})$ are the same as those in the corona theorem for $H^\infty(\mathbb{D})$.

2. The Bass stable rank of $A_S(\mathbb{D})$ is 1.

3. The topological stable rank of $A_S(\mathbb{D})$ is 2.

4. The ring $A_S(\mathbb{D})$ is coherent iff $S = \emptyset$.

The classes $A_S(\mathbb{D})$ serve as appropriate transfer function classes for infinite-dimensional systems that are not exponentially stable, but stable only in some weaker sense. Consequences of the above properties to stabilizing controller synthesis using a coprime factorization approach are discussed.

Bergman analytic spaces and measures

LUIS MANUEL TOVAR

Instituto Politécnico Nacional, Mexico

tovar@esfm.ipn.mx

In this work we deal with Bergman $F(p, q, s)$ spaces. Although actually these spaces have been already worked by R. Zhao and J. Rättyä, their results don't cover all the possibilities for the parameters p, q, s . Besides, with the help of a special measure we obtain several results and a complete metric on these spaces.

Approximation in Morrey spaces

N. PINAR TOZMAN

Balıkesir University, Turkey

ptuzkaya@yahoo.com

Coauthor: D. M. Israfilov

Let Γ be a rectifiable Jordan curve in \mathbb{C} and $\mathbb{T} := \{w : |w| = 1\}$. For $\alpha \in [0, 2]$ and $p \geq 1$, we define the Morrey space $L^{p,\alpha}(\Gamma)$ as the set of functions $f \in L^p_{loc}(\Gamma)$ such that

$$\|f\|_{L^{p,\alpha}(\Gamma)}^p := \sup_{B \cap \Gamma} \frac{1}{|B \cap \Gamma|^{1-\frac{\alpha}{2}}} \int_{B \cap \Gamma} |f(z)|^p |dz| < \infty,$$

where the supremum is taken over all balls of \mathbb{C} . It is a Banach space; for $\alpha = 2$ it coincides with $L^p(\Gamma)$ and for $\alpha = 0$ with $L^\infty(\Gamma)$. For a given function $f \in L^{p,\alpha}(\mathbb{T})$ and $r = 1, 2, \dots$, we define the r -modulus of smoothness $\omega_p^r(f, t) := \sup_{|h| \leq t} \|\Delta_h^r(f, \theta)\|_{L^{p,\alpha}(\mathbb{T})}$, where $\Delta_h^r(f, \theta) := \sum_{k=0}^r \binom{r}{k} (-1)^{r+k} f(\theta + kh)$, $h > 0$. For $f \in L^{p,\alpha}(\mathbb{T})$, we denote by $E_k(f)_{L^{p,\alpha}(\mathbb{T})} := \inf \{\|f - t_k\|_{L^{p,\alpha}(\mathbb{T})} : t_k \text{ is a polynomial of degree } \leq k\}$ the minimal error of approximation of f by trigonometric polynomials of degree at most k .

Problems of approximation theory in Smirnov and Lebesgue spaces defined in a set of \mathbb{C} were studied by many authors. In this talk we discuss the problems of approximation theory in the Morrey spaces $L^{p,\alpha}(\mathbb{T})$. Our main result is the following inverse theorem.

Theorem. Let $f \in L^{p,\alpha}(\mathbb{T})$, $\alpha \in [0, 2]$ and $p \geq 1$. Then for every $r = 1, 2, \dots$, there is a constant $c_r > 0$ such that $\omega_{p,\alpha}^r(f, 1/n) \leq c_r n^{-r} \sum_{k=1}^n k^{r-1} E_k(f)_{L^{p,\alpha}(\mathbb{T})}$ for $n = 1, 2, \dots$.

An extremal function for the multiplier algebra of the universal Pick space

FRANK WIKSTRÖM

Mid Sweden University, Sundsvall, Sweden

Frank.Wikstrom@miun.se

We will discuss a natural extremal function for the multiplier algebra of the universal Pick space and investigate its properties. This extremal function has connections to some naturally occurring extremal functions in pluripotential theory, in particular the pluricomplex Green function. We will discuss these connections, presenting several explicit examples.

Superposition operators on Bloch-type spaces

WEN XU

University of Joensuu, Finland

wxu@cc.joensuu.fi

In this paper we characterize all entire functions that transform a Bloch-type space into another by superposition in terms of their order and type or the degree of polynomials. We also prove that all superposition operators induced by such entire functions are bounded.

General Toeplitz operators on the analytic Besov spaces

NINA ZORBOSKA

University of Manitoba, Winnipeg, Canada

zorbosk@cc.umanitoba.ca

Coauthors: Z. Wu & R. Zhao

We present a characterization of complex measures μ on the unit disk for which the general Toeplitz operator T_μ^α is bounded or compact on the analytic Besov spaces B_p with $1 \leq p < \infty$.

2 Clifford and Quaternion Analysis

Organizers

Irene M. Sabadini (*Politecnico di Milano, Italy*)

Michael Shapiro (*Instituto Politécnico Nacional, Mexico*)

Frank Sommen (*Ghent University, Belgium*)

Clifford analytic methods for time-dependent equations

PAULA CEREJEIRAS

Universidade de Aveiro, Portugal

pceres@mat.ua.pt

Clifford analytic methods are particular adequate for the study of elliptic partial differential equations of mathematical physics since they provide a factorization of the second order partial operators in terms of appropriate Dirac operators and thus, orthogonal decompositions of function spaces where one of this spaces is the space of null-solutions of the Dirac operator.

In this talk we present a factorization of linear time-dependent equations in terms of parabolic Dirac operators depending on a convenient Witt basis. We will discuss its applications to the heat and to the Schrödinger operator.

The Cauchy integral in hermitian Clifford analysis

HENNI DE SCHEPPER

Ghent University, Belgium

hds@cage.ugent.be

Coauthors: F. Brackx & B. De Knock

Hermitian Clifford analysis has recently emerged as a refinement of the orthogonal case; it focuses on the simultaneous null solutions of two complex hermitian Dirac operators $\partial_{\underline{Z}}$ and $\partial_{\underline{Z}^\dagger}$ which decompose the Laplace operator as

$$4(\partial_{\underline{Z}}\partial_{\underline{Z}^\dagger} + \partial_{\underline{Z}^\dagger}\partial_{\underline{Z}}) = \Delta_{2n}$$

and which are invariant under the action of the unitary group. The study of complex Dirac operators as well as a systematic development of the associated function theory including the invariance properties with respect to the underlying Lie groups and Lie algebras is still in full progress. Naturally a Cauchy integral formula for hermitian monogenic functions taking values in the complex Clifford algebra \mathbb{C}_{2n} is essential to develop this function theory.

In this contribution it will be shown that a matrix approach is the key to obtain the desired result. Moreover, the hermitian Cauchy integral formula obtained reduces to the traditional Martinelli-Bochner formula for holomorphic functions of several complex variables in the special case of functions with values in the n -homogeneous part \mathbb{S}_n of the complex spinor space. This also means that the theory of hermitian monogenic functions not only refines orthogonal Clifford analysis, but also encompasses some important results of multidimensional complex analysis as special cases.

Extensions of Cauchy-type theorems in hyperbolic function theory

SIRKKA-LIISA ERIKSSON

Tampere University of Technology, Finland

sirkka-liisa.eriksson@tut.fi

Coauthor: H. Leutwiler

The aim of this talk is to consider the hyperbolic version of the standard Clifford analysis. The need for such a modification arises when one wants to make sure that the power function x^m is included. The leading idea is that the power function is the conjugate gradient of a harmonic function, defined with respect to the hyperbolic metric of the upper half space. The integral formula in the upper half space was proved in 2004. We consider the question how to extend this formula to the whole space and related results.

A new theory of regular functions of a quaternionic variable

GRAZIANO GENTILI

Università di Firenze, Italy

gentili@math.unifi.it

Coauthor: D. C. Struppa

In this talk we present the fundamental elements and results of a new theory of regular functions of one quaternionic variable. The theory we describe starts from a classical idea of Cullen, but we use a more geometric and general formulation to show that it is possible to build a rather complete theory. What we find is intriguing because, among other things, it allows the study of natural power series (and polynomials) with quaternionic coefficients, which is excluded when the Fueter approach is followed. We are also able to extend some important results for polynomials in the quaternionic variable to the case of power series.

Monogenic signals—sampling and approximation

UWE KÄHLER

Universidade de Aveiro, Portugal

uwek@mat.ua.pt

In recent years one can observe a growing interest in applications of Clifford analysis to signal processing, in particular to analytic signals. This interest is mainly driven by applications to texture analysis and to the mathematical foundations of the Huang-Hilbert transform. In this talk we will take a look at the definition and properties of monogenic signals in the case of the unit ball. Special emphasize will be given to sampling and frame properties.

Differential equations in algebras

YAKOV KRASNOV

Bar-Ilan University, Ramat-Gan, Israel

krasnov@math.biu.ac.il

We establish the basic qualitative properties of the solution to (non)linear DE's in algebras via arrangement their Pierce numbers. We study the influence of the Pierce numbers allocation on the stability theory of the nonlinear (quadratic and cubic) ODE's. As an application we demonstrate examples of DEs in Clifford algebras.

Applications of the Penrose transform in the theory of several Clifford variables

LUKÁŠ KRUMP

Karlovy University, Prague, Czech Republic

krump@karlin.mff.cuni.cz

For many years, there is a constant interest in understanding a structure of a resolution starting with the Dirac operator in several Clifford variables. The case of dimension 4 (the Fueter operator in several variables) is completely understood. In higher dimensions, the progress is still slow. The Penrose transform was successfully applied in dimension 4 and for two variables in higher dimensions. We shall show that the Penrose transform methods can be applied (in higher dimensions) also for more than two variables.

Infinite products in Clifford analysis

GUY LAVILLE

Université de Caen, France

guy.laville@math.unicaen.fr

In the classical theory of analytic functions in one complex variable, the main tool for the study of zeros is the infinite product. With it, explicit formulas for functions with prescribed zeros are possible. Is it possible to built such a theory in Clifford analysis? Of course, a product in this non-commutative context is not easy, but possible. We present a path leading to an infinite product. It gives explicit “regular functions” (monogenic and holomorphic cliffordian functions).

About the two-dimensional directional derivative of quaternionic functions

MARCO A. MACÍAS-CEDENO

Instituto Tecnológico y de Estudios Superiores de Monterrey, Mexico

marco.macias@itesm.mx

Coauthors: M. E. Luna-Elizarrarás & M. Shapiro

There will be presented the notion of the two-dimensional directional derivative of a quaternionic function which plays a role similar to that of the (inevitably one-dimensional!) directional derivative in the theory of complex functions. It is based on the idea of having an increment of the quaternionic variable “caught” in a two-dimensional plane, the latter being endowed with a complex structure inherited from the global quaternionic structure; it proved to be, in some occasions, a rather non-trivial task to introduce such a complex structure. The basic properties of the two-dimensional directional derivative will be explained, in particular, its relations with the holomorphy induced in the corresponding plane. What is more, the new notion proves to be intimately related with hyperholomorphic quaternionic function theory as well as with the theory of Cullen-regular quaternionic functions developed by Gentili and Struppa.

Cauchy transform on nonrectifiable surfaces in Clifford analysis

TANIA MORENO-GARCÍA

Universidad de Huelva, Cuba

tmorenog@facinf.uho.edu.cu

Coauthors: R. Abreu-Blaya & J. Bory-Reyes

We shall be concerned with relations of analytic properties of the multidimensional Cauchy transform

$$(Cu)(x) := \int_{\partial\Omega} \frac{\overline{y-x}}{\sigma_n |y-x|^{n+1}} \mathbf{n}(y) u(y) dy, \quad x \notin \partial\Omega$$

to the geometry of the corresponding domain Ω in the Euclidean space \mathbb{R}^{n+1} . The problem of determining for which pair $(\partial\Omega, u)$ the Cauchy transform Cu has continuous extension is studied for general open sets Ω without any *a priori* smoothness restrictions on the boundary. Our results are useful in connection with the problem of reconstructing a monogenic Clifford algebra-valued function from a prescribed jump condition across a nonrectifiable surface.

A note on Fueter's theorem

DIXAN PEÑA-PEÑA

Ghent University, Belgium

dixan@cage.ugent.be

Coauthor: F. Sommen

Assume f to be holomorphic in an open set of the upper half of the complex plane and put $f(z) = u(x, y) + iv(x, y)$ ($z = x + iy$), where as usual $u = \operatorname{Re} f$, $v = \operatorname{Im} f$. Then Fueter's theorem asserts that in the corresponding region the function

$$\Delta \left(u(q_0, |\underline{q}|) + \frac{q}{|\underline{q}|} v(q_0, |\underline{q}|) \right),$$

where $\underline{q} = q_1 i + q_2 j + q_3 k$ is a pure quaternion and $\Delta = \partial_{q_0}^2 + \partial_{q_1}^2 + \partial_{q_2}^2 + \partial_{q_3}^2$ the Laplace operator in four dimensional space, is monogenic with respect to the quaternionic Cauchy-Riemann operator $D = \partial_{q_0} + i\partial_{q_1} + j\partial_{q_2} + k\partial_{q_3}$.

Fueter's theorem is further generalized in a Clifford analysis setting. In this talk, we will discuss a new result which contains previous generalizations as special cases.

Dirichlet problem for pluriholomorphic functions of two complex variables

ALESSANDRO PEROTTI

University of Trento, Italy

perotti@science.unitn.it

The talk discusses the Dirichlet problem for pluriholomorphic functions of two complex variables. Pluriholomorphic functions are solutions of the system $\frac{\partial^2 g}{\partial \bar{z}_i \partial \bar{z}_j} = 0$ for every i, j . The Dirichlet problem for this system is not well posed and the homogeneous problem has infinitely many independent solutions.

The key point is the relation between pluriholomorphic and pluriharmonic functions. The link is constituted by the Fueter-regular functions of one quaternionic variable. Previous results about the boundary values of pluriharmonic functions and new results on L^2 traces of regular functions are applied to obtain a characterization of the traces of pluriholomorphic functions.

Double layer potential and Hilbert transform

TAO QIAN

University of Macau, China

fsttq@umac.mo

Through a double layer potential argument we define Hilbert transform on Lipschitz surfaces. The corresponding existence and boundedness of the transforms are proved. The related aspects of Hardy space are investigated.

Some properties of hypermonogenic functions in Clifford analysis

YUYING QIAO

Hebei Normal University, Shijihuang, China

yuyingqiao@163.com

In Clifford analysis hypermonogenic function is a kind of function which is the generalization of holomorphic function with the hyperbolic metric. The first part of this paper gives some properties of Hypermonogenic Function. The second part of the paper is Cauchy integral formula and Plemelj formula of bihypermonogenic function. And the last part of the paper is Cauchy integral formula and Plemelj formula of hypermonogenic function on unbounded domain.

Invariant syzygies for systems of hermitian Dirac operators

IRENE M. SABADINI

Politecnico di Milano, Italy

irene.sabadini@polimi.it

In this talk we describe the algebraic analysis of the system of differential equations described by the Hermitian Dirac operator, which is a linear first order operator invariant with respect to the action of the unitary group. We show that it is possible to give explicit formulae for the first syzygies of the resolution associated to the system, and we compute the number of minimal generators, both in the case of one and of several vector variables and, finally, we study the removability of compact singularities. We will also discuss the quaternionic version of the hermitian system.

Howe duality for Rarita-Schwinger representation

DALIBOR ŠMID

Karlovy University, Prague, Czech Republic

dalibor.smid@gmail.com

It is well known that the algebra of polynomials can be decomposed as a tensor product of harmonic polynomials and the set of invariants, which is generated by symbols of powers of Laplace operator (Fisher decomposition). An analogous decomposition is available for spinor-valued polynomials, where Dirac operator replaces Laplace. We study the case of polynomials valued in Rarita-Schwinger representation, where the structure of invariants is richer and may lead to a new example of Howe duality. As usually, the action of the Howe dual pair can clarify quite a few issues connected with a study of properties of solutions of R-S equations.

Micro-localization in Clifford analysis

FRANK SOMMEN

Ghent University, Belgium

fs@cage.ugent.be

The classical micro-localisation of the Dirac-delta distribution involves Radon decompositions in terms of curved or distorted plane waves with singular support in the origin and in one single direction so that the micro-support of these plane waves consists of just one point. By convolving functions or distributions with these waves it is then possible to decompose singularities, which is important for the description of earthquakes. Independent from this we obtained a Radon decomposition of the Cauchy kernel in terms of monogenic plane waves, a result which leads back to the classical Radon decomposition of the Dirac-delta distribution by taking boundary values of the form $f(x+0) - f(x-0)$ of monogenic functions. In our presentation we show that the micro-localisation formula of P. Lebeau may also be derived from the Radon decomposition of the Cauchy kernel by restricting this formula to the parabola $x_0 = |x|^2$ in the upper half space and multiplying the result with a suitable smooth function. The scalar part of the formula thus obtained is the classical micro-localisation formula; the Clifford analysis formula also contains a bivector part and may be used for the description of multivector earthquakes.

The initial value problem in Clifford and quaternion analysis

LE HUNG SON

Hanoi University of Technology, Vietnam

sonlehung2003@yahoo.com

Coauthors: N. Q. Hung & N. T. Van

The classical Cauchy-Kovalevskaya theory for holomorphic functions of one complex variable can be generalized in various directions. This talk deals with one such generalization in Clifford and quaternion analysis. For this purpose we consider the following initial value problem (IVP):

$$\frac{\partial u}{\partial t} = L \left[t, x, \frac{\partial u}{\partial x} \right], \quad u(0, x) = \phi(x),$$

where u is a regular function of the space variable x and the time variable t and taking values in a Clifford or quaternion algebra, and L is a partial differential operator of first order.

Theorem 1. Suppose that the operator L is associated to the Cauchy-Riemann operator in Clifford or quaternion analysis and the initial function $\phi(x)$ is an arbitrary regular function. Then the IVP is uniquely solvable. The solution $u(t, x)$ is regular for each t .

If L is partial differential operator of Vekua type and D is the Dirac operator in quaternion analysis, then under some assumptions on the coefficients of L we get our next result.

Theorem 2. The IVP is uniquely solvable and the solution $u(t, x)$ satisfies the side condition $Du = 0$ for each t .

Measures and function spaces

LUIS MANUEL TOVAR

Instituto Politécnico Nacional, Mexico

tovar@esfm.ipn.mx

By using an elementary Chebyshev inequality related with the measure of growth of a hyperholomorphic function, we introduce a new kind of weighted function space that preserves the main properties of classical analytic and hyperholomorphic function spaces (Bloch, D_p , Q_p).

Some rigidity results for regular maps

FABIO VLACCI

Università di Firenze, Florence, Italy

vlacci@math.unifi.it

As it is well known, for holomorphic functions (in one and in several complex variables) very elementary geometric conditions can imply strong analytic consequences. These phenomena are essentially connected with the definition of holomorphic map, so that they are commonly accepted and described as rigidities in the theory of holomorphic mappings. The aim of this talk is to give an overview on some recent developments in the subject with particular attention to boundary rigidity conditions which turn out to have a geometric and analytic interpretation. Finally we will show how some of the boundary rigidity conditions found for holomorphic mappings can be extended (or not ...) to the rich and new class of regular functions over Hamilton or Cayley numbers as recently introduced by G. Gentili and D. Struppa; these functions have good relations with complex holomorphic mappings and seem to become a very challenging new approach in the theory of quaternionic analysis.

Some integral representation formulas for functions with values in $C(V_{3,3})$

ZHONGXIANG ZHANG

Wuhan University, China

zhangzx9@sohu.com

In this paper, by using the solution of the Helmholtz equation $\Delta u - |h|^2 u = 0$, where $h = \sum_{i=1}^3 h_i e_i$, and the generalized Stokes formula, we firstly construct the explicit expressions of the kernel functions and then get the explicit integral representation formulas for functions with values in $C(V_{3,3})$. These integral representation formulas will play important role in studying the further properties of the functions with values in $C(V_{3,3})$.

3 Complex Analysis and Potential Theory

Organizers

Tahir Azeroglu Aliyev (*Gebze Institute of Technology, Turkey*)

Massimo Lanza de Cristoforis (*Università di Padova, Italy*)

Promarz M. Tamrazov (*National Academy of Sciences, Kiev, Ukraine*)

Multi-term asymptotic representation of the Riesz measure of subharmonic functions

POLINA AGRANOVICH

Verkin Institute for Low Temperature Physics and Engineering, Kharkiv, Ukraine

agranovich@ilt.kharkov.ua

One of the most important problems in the function theory is a question of the connection between the regularity in the distribution of zeros (masses) of an entire (subharmonic) function and the behavior of itself at infinity.

In the thirties of the previous century B. Levin and A. Pflüger simultaneously and independently constructed the theory of functions of completely regular growth which describes this connection in the terms of one-term asymptotics. But sometimes either the behavior of a subharmonic function or the growth of its measure of Riesz are given by multi-term asymptotic representation.

The investigation has shown that there are the essential differences in the behavior of the first term and the other terms of the asymptotics. So it was established that the first term of the Riesz measure asymptotics is the monotone non-decreasing function with respect to an angle variable for any fixed modulus. In the same time the second and the next terms of the asymptotics can cease to be functions of bounded variation. Thus it is natural to consider the problem about the influence of the reminder term on the properties of the main terms of the asymptotics. This question is study in this work.

Modified integral operators preserving subordination and superordination

TEODOR BULBOACĂ

Babeş-Bolyai University, Cluj-Napoca, Romania

bulboaca@math.ubbcluj.ro

Let $H(U)$ be the space of analytic functions in the unit disk U . For the integral operator $A_{\alpha, \beta, \gamma}^{\phi, \varphi} : \mathcal{K} \rightarrow H(U)$, with $\mathcal{K} \subset H(U)$, defined by

$$A_{\alpha, \beta, \gamma}^{\phi, \varphi}[f](z) = \left[\frac{\beta + \gamma}{z^\gamma \phi(z)} \int_0^z f^\alpha(t) \varphi(t) t^{\delta-1} dt \right]^{1/\beta},$$

where $\alpha, \beta, \gamma, \delta \in \mathbb{C}$ and $\phi, \varphi \in H(U)$, we determine sufficient conditions on g_1, g_2, α, β and γ such that $z\varphi(z)[g_1(z)/z]^\alpha \prec z\varphi(z)[f(z)/z]^\alpha \prec z\varphi(z)[g_2(z)/z]^\alpha$ implies

$$z\phi(z) \left[A_{\alpha, \beta, \gamma}^{\phi, \varphi}[g_1](z)/z \right]^\beta \prec z\phi(z) \left[A_{\alpha, \beta, \gamma}^{\phi, \varphi}[f](z)/z \right]^\beta \prec z\phi(z) \left[A_{\alpha, \beta, \gamma}^{\phi, \varphi}[g_2](z)/z \right]^\beta.$$

The symbol “ \prec ” stands for subordination, and we call such a result a *sandwich-type theorem*.

In addition, $z\phi(z)[A_{\alpha, \beta, \gamma}^{\phi, \varphi}[g_1](z)/z]^\beta$ is the *largest* function and $z\phi(z)[A_{\alpha, \beta, \gamma}^{\phi, \varphi}[g_2](z)/z]^\beta$ is the *smallest* function so that the left-hand side, respectively the right-hand side of the above implication hold, for all functions f satisfying the differential subordination, respectively the differential superordination of the assumption.

We give particular cases of the main result obtained for appropriate choices of ϕ and φ that also generalize classic results of the theory of differential subordination and superordination.

A new generalization of the scalar Poisson kernel in two dimensions

SERAP BULUT

Balıkesir University, Turkey

sbulut@balikesir.edu.tr

In this work we give a new generalization of the scalar Poisson kernel in two dimensions and discuss an integral formula for this. Set

$$S(\theta; x, y, z, t, u, v) = \frac{L(x, y, z, t, u, v)}{(1 - xe^{i\theta})(1 - ye^{-i\theta})(1 - ze^{i\theta})(1 - te^{-i\theta})(1 - ue^{i\theta})(1 - ve^{-i\theta})},$$

where x, y, z, t, u, v are complex parameters all with modulus less than 1,

$$L(x, y, z, t, u, v) = \frac{(1 - xy)(1 - xt)(1 - xv)(1 - yz)(1 - yu)(1 - zt)(1 - zv)(1 - tu)(1 - uv)}{K(x, y, z, t, u, v)},$$

$$\begin{aligned} K(x, y, z, t, u, v) = & 1 - [xz + xu + zu][yt + yv + tv] + xzu[y^2(t + v) + t^2(y + v) + v^2(y + t)] \\ & + ytv[x^2(z + u) + z^2(x + u) + u^2(x + z)] \\ & - [x^2zu + xz^2u + xzu^2][y^2tv + yt^2v + ytv^2] + 4xyztuv + x^2y^2z^2t^2u^2v^2. \end{aligned}$$

Theorem. $\frac{1}{2\pi} \int_0^{2\pi} S(\theta; x, y, z, t, u, v) d\theta = 1.$

Real analytic dependence of the single and double layer potentials of some constant-coefficient elliptic partial differential operators upon perturbation of the density, the support and the coefficients of the operator

MATTEO DALLA RIVA

Università di Padova, Italy

mdallari@math.unipd.it

Coauthor: M. Lanza de Cristoforis

We introduce a particular single layer potential of a given constant coefficient elliptic partial differential operator of order $2k$. Then, in the frame of Schauder spaces, we show a real analyticity result for the dependence of such a potential and its derivatives till order $2k - 1$ upon suitable perturbations of the domain, the coefficients of the operator and of the density. Exploiting such a result, we consider the dependence upon perturbations of the domain, the coefficients and the density of the single and double layer potentials which arise in certain boundary value problems, such as the Dirichlet and Neumann problems for the Lamé equations and the Stokes system. We show also in this case that the dependence is real analytic.

The geometry of Blaschke product mappings

DORIN GHISA

York University, Toronto, Canada

dghisa@yorku.ca

Coauthor: I. Barza

A Blaschke product B generates a covering Riemann surface (W, B) of the complex plane. The study of such surfaces is undertaken here in a very general context, namely when the cluster points of the zeros of B is a (generalized) Cantor set. Explicit forms and fundamental domains for the covering transformations are revealed.

On (α, Q) -homeomorphisms

ANATOLY GOLBERG

Bar-Ilan University, Ramat-Gan, Israel

golberga@hit.ac.il

The quasi-invariance of conformal module is one of characteristic properties of quasiconformality. Martio, Ryazanov, Srebro, and Yakubov (2004) have introduced a notion of Q -homeomorphisms, where Q is a measurable function in a domain $G \subset \mathbb{R}^n, n \geq 2$. A homeomorphism f is Q -homeomorphism, if the conformal module satisfies

$$M(f(\Gamma)) \leq \int_G Q(x) \rho^n(x) dx$$

for every curve family Γ in G and for every admissible function ρ for Γ .

In this talk, we consider (α, Q) -homeomorphisms, related to more general inequalities

$$M_\alpha(f(\mathcal{S}_k)) \leq \int_G Q(x) \rho^\alpha(x) dx.$$

Here $M_\alpha(\mathcal{S}_k)$ denotes the α -module of the family of k -dimensional surfaces in G ($1 \leq k \leq n-1$). We establish differential and geometric properties of (α, Q) -homeomorphisms f .

On characterization of the extension property

ALEXANDER GONCHAROV

Bilkent University, Ankara, Turkey

goncha@fen.bilkent.edu.tr

Given a compact set $K \subset \mathbb{R}$, let $\mathcal{E}(K)$ denote the space of Whitney jets on K , that is traces on K of functions from $C^\infty(\mathbb{R})$. The compact set K is said to have the extension property if there exists a linear continuous extension operator $L : \mathcal{E}(K) \rightarrow C^\infty(\mathbb{R})$. We suggest a geometric characterization of the extension property for the Cantor-type sets. In this case the criterion can also be given in terms of Potential Theory, namely in terms of the rate of growth of the values of the minimal discrete energy of compact sets that locally form K .

On convergence of Bieberbach polynomials in closed domains

DANIYAL M. ISRAFILOV

Balikesir University, Turkey

mdaniyal@balikesir.edu.tr

In this talk we discuss the uniform convergence of the Bieberbach polynomials which allow the possibility of a practical approximation of the Riemann conformal mapping on closed simply connected domains with various geometric properties, and then we estimate the rate of this convergence depending on the geometric properties of these domains.

On finite difference properties of conformal mappings

OLENA KARUPU

National Aviation University, Kyiv, Ukraine

karupu@ukr.net

Let a simply connected domain in the complex plane bounded by a smooth Jordan curve be given. We consider the angle between the tangent to the curve and the positive real axis as the function of the arc length on the curve. Suppose that a homeomorphism of the closed unit disk onto the closure of the domain considered that is conformal in the open unit disk is given.

O. Kellogg proved that if the angle between the tangent to the curve and the positive real axis satisfies a Hölder condition, then the derivative of the function realizing the conformal mapping satisfies the Hölder condition with the same index. Afterwards connections between properties of the boundary of the domain and properties of the function considered were investigated in works by several authors. In particular, generalizations of Kellogg-type theorems were obtained by the author in the terms of the uniform curvilinear, noncentralized local arithmetic and integral moduli of smoothness.

We consider some new finite difference properties of the function realizing the conformal mapping on the boundary of the domain formulated for general moduli of smoothness of arbitrary order.

The jump problem on non-rectifiable curves and metric dimensions

BORIS A. KATS

Kazan State Architecture and Building University, Russia

architec@mi.ru

Let Γ be a non-rectifiable closed Jordan curve on the complex plane \mathbb{C} . It divides the plane into two parts: a bounded domain D^+ and a domain D^- containing the point ∞ . We seek a function $\Phi(z)$ holomorphic in $\overline{\mathbb{C}} \setminus \Gamma$ such that $\Phi(\infty) = 0$, with boundary values $\Phi^+(t) := \lim_{D^+ \ni z \rightarrow t} \Phi(z)$ and $\Phi^-(t) := \lim_{D^- \ni z \rightarrow t} \Phi(z)$ for any $t \in \Gamma$, and

$$\Phi^+(t) - \Phi^-(t) = f(t), \quad t \in \Gamma,$$

where $f(t)$ is a known function defined on Γ . This boundary value problem is well known as the jump problem. If Γ is piecewise smooth, then its solution is given by the Cauchy integral.

In the present report we introduce a new metric characteristic of dimensional type for non-rectifiable curves and obtain new conditions of solvability of the jump problem on non-rectifiable curves in terms of this version of dimension.

The Riemann-Hilbert problem in weighted classes of Cauchy-type integrals with density in $L^{p(\cdot)}(\Gamma)$

VAKHTANG KOKILASHVILI

Razmazde Mathematical Institute, Tbilisi, Georgia

kokil@rmi.acnet.ge

Coauthor: V. Paatashvili

We consider the Riemann-Hilbert problem formulated as follows: define a function $\phi \in K^{p(\cdot)}(D; \omega)$ whose boundary values $\phi^+(t)$ satisfy the condition $\operatorname{Re}[(a(t) + ib(t))\phi^+(t)] = c(t)$ a.e. on Γ . Here D is the finite simply connected domain bounded by a simple closed curve Γ , and $K^{p(\cdot)}(D; \omega)$ is the set of functions $\phi(z)$ representable in the form $\phi(z) = \omega^{-1}(z)(K_\Gamma \varphi)(z)$, where $\omega(z)$ is a weight function and $(K_\Gamma \varphi)(z)$ is a Cauchy type integral whose density φ is integrable with a variable exponent $p(t)$. It is assumed that Γ is a piecewise-Lyapunov curve without zero angles, $\omega(z)$ is an arbitrary power function and $p(t)$ satisfies the log-Hölder condition. The solvability conditions are established and solutions are constructed. In addition to the weight ω and functions a , b , c , these solutions largely depend both on the values of $p(t)$ at the angular points of Γ and on the values of angles at these points.

Singular perturbation problems in potential theory: A functional analytic approach

MASSIMO LANZA DE CRISTOFORIS

Università di Padova, Italy

mldc@math.unipd.it

We consider an unperturbed domain \mathbb{I}^o of \mathbb{R}^n with $0 \in \mathbb{I}^o$, and a parameter $\epsilon > 0$, and another domain \mathbb{I}^i of \mathbb{R}^n containing 0, and we remove from \mathbb{I}^o the domain $\epsilon\overline{\mathbb{I}^i}$, thus obtaining the annular domain $\mathbb{A}(\epsilon) \equiv \mathbb{I}^o \setminus \epsilon\overline{\mathbb{I}^i}$. Then we consider a boundary value problem on $\mathbb{A}(\epsilon)$ admitting unique solution for all $\epsilon > 0$ sufficiently small, and we question how the solution of the problem in $\mathbb{A}(\epsilon)$ or a suitable functional of such a solution behave as ϵ is close to 0. In this talk we present an approach which aims at representing the solution or a suitable functional of such a solution in terms of real analytic operators defined in a whole neighborhood of 0 and in terms of possibly singular but known functions of ϵ such as $\log \epsilon$, ϵ^{-1} , etc...

On some properties of the layer potentials for the Helmholtz equation treated with hypercomplex analysis

MARCO A. MACÍAS-CEDENO

Instituto Tecnológico y de Estudios Superiores de Monterrey, Mexico

marco.macias@itesm.mx

Coauthor: M. Shapiro

The harmonic single and double-layer potentials exist in any dimension but it is well known that in the plane they can be treated quite efficiently with methods of holomorphic function theory. At the same time their metaharmonic, or acoustic, counterparts are studied directly without any reference to function theory. This can be seen distinctly in D. Colton & R. Kress [*Integral Equation Methods in Scattering Theory*, Wiley, 1983] and R. Kress [*Linear Integral Equations*, Springer, 1989], and in others sources as well. The reason of such different approaches lies in the supposed absence of “complex analysis” for the Helmholtz equation. The goal of the talk is to explain that hyperholomorphic Cauchy-type integral for a certain version of quaternionic analysis in the plane is the exact replacement of the holomorphic Cauchy-type integral in the sense that it combines both respective potentials which allows to use widely an analogy with the plane harmonic case. A few known facts were chosen intentionally in order to demonstrate a deep similarity between both methods and thus a unifying role of quaternionic analysis.

Analogue of Jackson-Bernstein theorem in L_p on closed curves in the complex plane

JAMAL I. MAMEDKHANOV

Academy of Sciences, Baku, Azerbaijan

jamalmamedkhanov@rambler.ru

Coauthor: D. I. Dadashova

Jackson-Bernstein’s classical theorem, valid for periodic functions in the class $Lip_{(0,2\pi)}\alpha$ ($0 < \alpha < 1$) asserts that for $f \in Lip_{(0,2\pi)}\alpha$ it is necessary and sufficient that

$$E_n(f, [0, 2\pi]) = \inf_{P_n} \|f - P_n\|_{C[0,2\pi]} \leq const \cdot n^{-\alpha}.$$

This work is devoted to Jackson-Bernstein’s theorem on closed curves in the complex plane in the metrics $L_p(\Gamma)$. To this end we prove a sufficiently general theorem of Jackson-Bernstein type on sufficiently general classes of curves in the complex plane.

Let $p \geq 1$. Let the function $u(z)$ be given on some closed rectifiable curve Γ and $L_p(\Gamma, u)$ mean the weight space of functions f with weight u and with norm $\|f\|_{L_p(\Gamma, u)} := \|uf\|_{L_p(\Gamma)}$.

Let's denote by $E_p^*(G)$ the class of functions $f \in E_p(G)$ with finite modulus of smoothness $\omega_p^{(2)}(f, \delta)_\Gamma = \delta^2 \sup_{t \geq \delta} t^{-2} \sup_{|h| \leq t} \|f(z_h) + f(z_{-h}) - 2f(z)\|_{L_p(\Gamma)}$. We say that a Jordan rectifiable

curve Γ belongs to class M if for each $f \in M(\overline{G})$ the integral $(JF_h)_\Gamma(t) = \frac{1}{\pi i} \int_\Gamma \frac{F_h(z)}{z-t} dz$, $t \in \Gamma$, exists almost everywhere on Γ , and the estimate $\|JF_h\|_{L_p(\Gamma)} \leq C(p)\|F_h\|_{L_p(\Gamma)}$ holds for all $h \in [0, \pi]$, where $C(p)$ is a constant depending only on p .

Theorem. If $\Gamma \in M$ and $f \in E_p^*(G)$, then for each natural n there exists a polynomial P_n such that $\|f - P_n\|_{L_p(\Gamma)} \leq \text{const } \omega_p^{(2)}(f, 1/n)_\Gamma$.

Conformal mappings and Julia polynomials

BURÇİN OKTAY

Balıkesir University, Turkey

burcino78@hotmail.com

Conformal mappings play an important role in applied mathematics and physics. But it is not easy to find their explicit expressions. For this reason, the approximation problems to these mappings by the functions whose explicit expressions are known arise. In this way, the properties of conformal mappings can be estimated. As the functions which approximate to conformal mappings, generally polynomials are used.

In this talk, the approximation problems to these functions by the Julia polynomials which are a solution of an extremal problem are given and the approximation properties of these polynomials are investigated in some complex domains.

Commutative algebras and spatial potential fields

SERGIY PLAKSA

National Academy of Sciences, Kiev, Ukraine

plaksa@imath.kiev.ua

Analytic function methods in the complex plane for plane potential fields inspire searching of analogous methods for spatial potential solenoid fields.

Apparently, W. Hamilton made the first attempts to construct an algebra associated with the three-dimensional Laplace equation in the sense that the components of differentiable functions taking values in this algebra satisfy the Laplace equation. However, after constructing the quaternion algebra he had not studied a problem about constructing any other algebra.

I. Mel'nichenko (1975) considered the problem to construct a commutative associative Banach algebra such that monogenic (i.e. differentiable according to Gateaux) functions taking values in this algebra have components satisfying the three-dimensional Laplace equation.

We consider an infinite-dimensional commutative Banach algebra \mathbf{F} over the field of real numbers and establish that any spherical function is a component of some monogenic function taking values in this algebra. Thus monogenic functions taking values in \mathbf{F} form the widest of known functional algebras associated with the three-dimensional Laplace equation.

Grötzsch's problem

PROMARZ M. TAMRAZOV

National Academy of Sciences, Kiev, Ukraine

proma@imath.kiev.ua

We study the following problem: *Among all continua containing the fixed finite point collection belonging to the unit disk, find one that has the minimal hyperbolic capacity.*

This problem was formulated and discussed by H. Grötzsch (1930) as the hyperbolic analog of the Tchebotaröv's problem (the latter being posed in 1925).

Sufficient conditions for starlikeness and convexity of an analytic function

NIKOLA TUNESKI

Cyril and Methodius University, Skopje, Macedonia

nikolat@mf.edu.mk

Coauthor: E. Georgieva-Celakoska

Let \mathcal{A} be the class of analytic functions in the unit disk normalized with $f(0) = f'(0) - 1 = 0$. In this work the theory of differential subordinations is used for studying linear combinations of $f(z)/z$, $f'(z)$ and $f''(z)$ for $f \in \mathcal{A}$, and obtaining sufficient conditions for starlikeness and convexity. Comparison with previously known results is given.

Riemann boundary value problem on an open rectifiable curve

JULIA V. VASILIEVA

National Academy of Sciences, Kiev, Ukraine

kudjavina@mail.ru

Let γ be an oriented open Jordan rectifiable curve in the complex plane \mathbb{C} from a_1 to a_2 . Let $T := \{a_1, a_2\}$. By H_T we denote the set of all holomorphic functions Φ in $\mathbb{C} \setminus \gamma$ which have a limit in infinity and limiting values $\Phi^+(t)$ and $\Phi^-(t)$ in points $t \in \gamma \setminus T$ from the left and right of γ , respectively, and satisfy the condition $|\Phi(z)| \leq c \sum_{j=1}^2 |z - a_j|^{-\nu_F}$ for all $z \in \mathbb{C} \setminus \gamma$ with $0 < \nu_F < 1$. Consider the Riemann boundary value problem of finding a function $\Phi \in H_T$ such that the limiting values Φ^+ and Φ^- satisfy $\Phi^+(t) = G(t)\Phi^-(t) + g(t)$ for all $t \in \gamma \setminus T$, where G and g are given functions.

The solvability of the homogeneous Riemann boundary value problem on an arbitrary γ is obtained with the help of corresponding result of [Y. V. Vesileva & S. A. Plaksa, *Ukrainian Math. J.* **58** (2006), 694–708] and described in classical form. At the same time the non-homogeneous Riemann boundary value problem is considered on open curves of more general form than in the papers by R. K. Seifullaev (1980), K. Kutlu (2000), D. Peña-Peña & J. Bory (2002), and under this we use some additional (in comparison with the case of the closed curve) assumptions on γ . The solvability of the non-homogeneous problem is described in classical form in the case of finite index and under some minimal assumptions on G and g .

Geometry of \mathbb{C} -convex sets

YURI B. ZELINSKII

National Academy of Sciences, Kiev, Ukraine

zel@imath.kiev.ua

A set $E \subset \mathbb{C}^n$ is called linearly convex if for every $z \in \mathbb{C}^n \setminus E$ there is a hyperplane l such that $z \in l \subset \mathbb{C}^n \setminus E$. A set $E \subset \mathbb{C}^n$ is called locally linearly convex if for every $z \in \mathbb{C}^n \setminus E$ there is a hyperplane l and a neighborhood $U(z)$ such that $z \in l \cap U(z)$ and $l \cap E \cap U(z) = \emptyset$. A set $E \subset \mathbb{C}^n$ is called \mathbb{C} -convex if for every complex line γ the sets $\gamma \cap E$ and $\gamma \setminus \gamma \cap E$ are connected.

Theorem 1. If $D = D_1 \times D_2$ is a bounded \mathbb{C} -convex domain, then D_j , $j = 1, 2$ are convex.

Theorem 2. Let $D \subset \mathbb{C}^n$ be a locally linearly convex domain with C^2 boundary. Then

- (a) D is C -convex and homeomorphic to the unit ball if ∂D is connected and $\partial_{\mathbb{C}P^n} D = \overline{\partial_{\mathbb{C}^n} D}$;
- (b) $D = D_1 \times \mathbb{C}^{n-1}$, where D_1 is a subset of a complex line, if ∂D is not connected;
- (c) D is a projective image of $D_1 \times \mathbb{C}^{n-1}$, where $D = D_0 \setminus \text{one point}$ with D_0 simply connected if $\partial_{\mathbb{C}P^n} D \neq \overline{\partial_{\mathbb{C}^n} D}$.

Theorem 3. Let $A = \{A_i\}$ be a family of convex compacts in \mathbb{R}^n such that every k members of A have a common point. Then either every $k + 1$ members of A have a common point a , or there exists $k + 1$ compacts A'_i for which $H^{k-1}(\bigcup_{i=1}^{k+1} A'_i) \neq \emptyset$.

Theorem 4. Let $\{K_j\}$, $j = 1, \dots, l$ be a family of (n, m) -convex compacts, $l < \frac{n}{n-m}$ and its every $l - 1$ elements have a common point. Then either they all have a common point, or $H^i(\bigcup_1^l K_j) = \emptyset$ for $i > n - \frac{n}{n-m} + l - 2$.

4 Complex Analytic Methods for Applied Sciences

Organizers

Sergei V. Rogosin (*Belarus State University, Minsk, Belarus*)

Vladimir V. Mityushev (*Pedagogical University, Stupsk, Poland*)

Schwarz-Christoffel mappings for non-polygonal regions

ANDERS ANDERSSON

Växjö University, Sweden

`anders.andersson@vxu.se`

Assume that R is the upper half-plane or the unit circle, and that f is the Schwarz–Christoffel mapping from R to a polygonal region P in the complex plane.

For certain regions $Q \subset R$, for example concentric circles in the unit circle or sectors in the upper half-plane, we will discuss properties of the region $f(Q)$ and its boundary, especially concerning the curvature. Furthermore, given a region Ω , we will discuss methods to find a suitable polygon P and region Q , such that $f(Q) = \Omega$.

Functional-differential equations in Hardy-type classes

PIOTR DRYGAŚ

University of Rzeszow, Poland

`drygaspi@univ.rzeszow.pl`

We consider a conjugation problem for harmonic functions in multiply connected circular domains. This problem is rewritten in the form of the R -linear boundary value problem by using equivalent functional-differential equations in a class of analytic functions. It is proven that the operator corresponding to the functional-differential equations is compact in the Hardy-type space. Moreover, these equations can be solved by the method of successive approximations under some natural conditions. This problem has applications in mechanics of composites when the contact between different materials is imperfect.

Experimental modeling of surface roughness in facing process of CK45 steel

MEHDI ESKANDARZADE

University of Urmia, Iran

`m.eskandarzade@gmail.com`

Coauthors: A. Doniavi & H. Ebrahimi

The surface finish of machined parts is known to have considerable effect on some properties such as wear resistance and tolerance. In most recent research the surface quality of cutting process in turning is investigated, but facing is less attended. This paper investigates surface quality of facing in turning process of CK45 steel. Factorial design methodology is used to develop an empirical model that explains the relationship between three main parameters of cutting speed, feed rate and depth of cut. The result of ANOVA analysis shows that the first order of single effect for each parameter has significant effect on surface roughness. The results from predictive values are in a good agreement with actual data, which confirms the model.

Modeling of excitation force in electromagnetic acoustic transducer systems

MEHDI ESKANDARZADE

University of Urmia, Iran

`m.eskandarzade@gmail.com`

Coauthor: A. Doniavi

Electromagnetic acoustic transducers (EMATs) generate ultrasonic waves due to Lorentz and magnetostrictive forces. However, only the Lorentz mechanism is present in non-magnetic conducting materials. In most previous research the effect of dynamic magnetic field has been neglected, while it significantly influences the wave propagation phenomena in an EMAT system. In this research we have investigated the Lorentz type EMAT system and derived complete equations of ultrasonic wave excitation force. The equation developed shows that the dynamic component of total Lorentz force is in a different direction with that of the static magnetic field. And in some cases this type of force significantly influences the total Lorentz force.

Phenomenology test for shell models in a turbulence problem

IGOR M. GAISSINSKI

Technion, Haifa, Israel

`igorg@aerodyne.technion.ac.il`

Coauthor: V. Y. Rovenski

Nonlinear interactions between the basis functions used in the shell model are described by the tensor $T = \{T_{NML}\}$ obtained by using Galerkin projection of Navier-Stokes equations on the wavelet basis. The coefficients T_{NML} must satisfy the conservation laws for the energy and the enstrophy or the helicity (depending on dimension 2 or 3). We deduce these relations and present their solutions as a product of the partial solution and $G(\eta_{NML}, \eta_{LNM}, \eta_{MLN})$, where G is any odd function invariant under transpositions of arguments, and the tensor η has zero diagonal terms and satisfies the symmetries $\eta_{NML} = -\eta_{LNM}$, $\eta_{N+k, M+k, L+k} = \eta_{NML}$. One may use these relations and their solution to verify the adequacy of wavelet basis functions for the turbulence problem.

Multiparameter spectral theory of operator collections acting in different complex Banach spaces

MISIR B. RAGIMOV

Baku State University, Azerbaijan

`faig@bsu.az`

In this work, evidently, for the first time there is built multiparameter spectral theory of operators collections, acting in different complex Banach spaces. There is introduced and essentially used a number of new concepts: the left and right pseudoresolvents and there is built the left and right Banach algebras. At that there is used the technique of commutative Banach algebras and group representations. The obtained results give answers to a number of common questions of multi-parameter spectral theory of linear operators.

On a vector-matrix boundary value problem in a multiply connected circular domain

SERGEI V. ROGOSIN

Belarus State University, Minsk, Belarus

rogosin@gmail.com

Coauthor: M. V. Dubatovskaya

The heat conduction problem for a bounded symmetric 2D composite material was reduced recently (Dubatovskaya-Rogosin, 2006) to a mixed vector-matrix boundary value problem

$$\begin{aligned}\Phi^+(t) &= D\Phi^-(t), \quad t \in (0, 1), \\ \Phi^+(t) &= H_k\Phi^-(t) + S_k\overline{\Phi^-(t)}, \quad |t - a_k| = r, \quad a_k = r_0 e^{\frac{\pi(2k-1)}{6}}, \quad k = 1, \dots, 6, \\ \operatorname{Re} \Phi(t) &= q(t), \quad |t| = 1,\end{aligned}\tag{1}$$

with respect to a piecewise analytic vector, bounded in a neighborhood of the points $z = 0$, $z = 1$, and satisfying certain symmetry relations in the domain. Here D is a given diagonal matrix with equal constant diagonal elements, H_k , S_k are given constant matrices, q is a given vector-function. By solving problem (1), we reduce our mixed problem to a vector-matrix \mathbb{R} -linear conjugation problem in the bounded multiply connected circular domain $\{z \in \mathbb{C} : |z| < 1; |z - a_k| > r, k = 1, \dots, 6\}$. This problem is investigated by the method of functional equations as developed in the monograph by Mityushev and Rogosin (1999).

Periodic diffraction boundary-value problems and Toeplitz operators

ANA MOURA SANTOS

Instituto Superior Tecnico, Lisbon, Portugal

amoura@math.ist.utl.pt

Coauthors: M. A. Bastos & P. Lopes

We consider a wave diffraction problem by a periodic strip grating. Two boundary-value problems are studied in detail for an arbitrary geometry of the grating: the oblique derivative and the classic Neumann boundary-value problems. After we formulate these problems as convolution type operators acting on Bessel potential periodic spaces, associated operators acting on spaces of matrix functions defined on composed contours are derived. Within the last setting, Fredholm properties and Fredholm indices for the oblique derivative and the Neumann boundary-value problems are given.

Basic boundary value problems for homogeneous and inhomogeneous Cauchy-Riemann equations in a concentric ring domain

TATYANA S. VAITEKHOVICH

Belarus State University, Minsk, Belarus

vaiteakhovich@mail.ru

Four basic boundary value problems, namely Schwarz, Dirichlet, Neumann, and Robin problems are considered for analytic functions or, equivalently, to the homogeneous Cauchy-Riemann equation, in a concentric ring domain $R := \{0 < r < |z| < 1\}$ of the complex plane.

Contrary to the case of simply connected domains, here appears some solvability condition even for the Schwarz problem. The function $\log z$ shows that this condition is necessary for solving the problem. Similarly solvability conditions appear in all other cases. The Robin problem is only investigated in a particular form. More general ones related to analytic functions may be handled in similar ways.

The main tools to treat boundary value problems for complex first order partial differential equations are the Gauss theorem and Cauchy-Pompeiu integral representations. For our purpose the special representation formula for analytic functions in a circular ring domain is derived.

Boundary value problems for the inhomogeneous Cauchy-Riemann equation can be reduced to the ones for analytic functions using the Pompeiu operator. All solutions are represented and the solvability conditions are given in explicit forms via integral formulas.

5 Complex and Functional Analytic Methods in PDEs

Organizers

Heinrich Begehr (*Freie Universität, Berlin, Germany*)

Dao-Qing Dai (*Zhongshan University, Guangzhou, China*)

Jin Yuan Du (*Wuhan University, China*)

Neumann problem for a particular fourth-order equation

ÜMIT AKSOY

Atılım University, Ankara, Turkey

uaksoy@atilim.edu.tr

Coauthor: O. Çelebi

In this talk we investigate the solutions of the Neumann problem for

$$\Delta^2 w + \sum_{k+l \leq 4} q_{kl} \frac{\partial^{k+l} w}{\partial z^k \partial \bar{z}^l} = f(z)$$

in the unit disc. First we transform the differential equation to a singular integral equation. Then we use Fredholm theory to find the solution.

A new solution of some weighted problems for the Riemann-Liouville and Weyl operators

MUSTAFA AVCI

Dicle University, Diyarbakır, Turkey

mustafaxavci@hotmail.com

In this study, we introduce new proofs of the boundedness of the Riemann-Liouville transform $R_\alpha f(x) = \int_0^x (x-t)^{\alpha-1} f(t) dt$ and the Weyl transform $W_\alpha f(x) = \int_x^\infty (t-x)^{\alpha-1} f(t) dt$ for the weight functions v and ω from L_ω^p to L_v^q when $1 < p \leq q < \infty$, $1/p < \alpha < 1$ or $\alpha > 1$.

We denote the weighted Lebesgue space by $L_\omega^p(0, \infty)$ and its norm by $\|f\|_{L_\omega^p}$. Let

$$W(x) = \int_0^x \omega^{1-p'}(t) dt, \quad V(x) = \int_x^\infty \frac{v(t)}{t^{(1-\alpha)q}} dt, \quad \overline{W}(x) = \int_x^\infty \omega^{1-p'}(t) dt, \quad \overline{V}(x) = \int_0^x \frac{v(t)}{t^{(1-\alpha)q}} dt$$

where $p + p' = pp'$. Assume that there are $\eta_1, \delta_1, \eta_2, \delta_2 > 0$ such that $V(x/2) \leq 2^{\eta_1} V(x)$, $W(x/2) \leq 2^{-\delta_1} W(x)$, $\overline{V}(x/2) \leq 2^{-\delta_2} \overline{V}(x)$, and $\overline{W}(x/2) \leq 2^{\eta_2} \overline{W}(x)$ all for $x > 0$.

Theorem 1. The inequality $\|R_\alpha f\|_{L_v^q} \leq A \|f\|_{L_\omega^p}$, where the positive constant A does not depend on f , is fulfilled if and only if $B := \sup_{x>0} (V(x))^{1/q} (W(x))^{1/p'} < \infty$. Moreover, if A is the best constant, then $A \approx B$.

Theorem 2. The inequality $\|W_\alpha f\|_{L_v^q} \leq \overline{A} \|f\|_{L_\omega^p}$, where the positive constant \overline{A} does not depend on f , is fulfilled if and only if $\overline{B} := \sup_{x>0} (\overline{W}(x))^{1/p'} (\overline{V}(x))^{1/q} < \infty$. Moreover, if \overline{A} is the best constant, then $\overline{A} \approx \overline{B}$.

Complex partial differential equations

HEINRICH BEGEHR

Freie Universität, Berlin, Germany

begehr@math.fu-berlin.de

Boundary value problems for special complex equations as the inhomogeneous polyanalytic and the inhomogeneous polyharmonic equations are investigated. Starting from lower order equations, by iteration, problems for higher order equations are constructed in a natural way.

To the Cauchy-Riemann operator there are basic boundary value problems of Schwarz, Dirichlet, Neumann, and Robin type. Not all of them are well posed. But the related solvability conditions are available. For the Bitsadze operator hence one can prescribe any of these conditions for the function itself and its image under the Cauchy-Riemann operator. Similarly this is true also for the Laplace operator. Decomposing it in a system of a Cauchy-Riemann and an anti Cauchy-Riemann equation one can prescribe again such combinations of boundary conditions. But as is well known for the Poisson equation there are particular boundary value problems, the Dirichlet, the Neumann, and the Robin boundary value problems, prescribing only one rather than two conditions on the boundary. Hence there are two kinds of boundary value problems for higher order equations, some of which are decomposable in lower order ones and others which are not. One type consists of those which are attained from lower order ones others are not conform to this decomposition. This situation is true in particular for the polyharmonic equation. By an iteration process polyharmonic Green functions can be constructed which are of multiple type. In the biharmonic case there are Green-2, Neumann-2, Robin-2, hybrid Green-Neumann, hybrid Green-Robin, and hybrid Neumann-Robin functions. The *-2 functions are symmetric, but e.g. the hybrid Green-Neumann function behaves in one variable as a Green in the other as a Neumann function. But there is a biharmonic Green function not related to the ones listed. The situation becomes more involved for higher order polyharmonic operators because more and more lower order “Green” functions are available.

Particular results will be presented in the disc, the upper half plane and a quarter plane.

On a system consisting of equations of second and fourth orders

NATALIA CHINCHALADZE

Tbilisi State University, Georgia

natalic@viam.sci.tsu.ge

The aim of this talk is to study some systems consisting of equations of second and fourth orders connected to the elastic solid-fluid interection problems [N. Chinchaladze and G. Jaiani, *Appl. Math. Inform.* **6** (2001), 25–64]. The well-posedness of the interection problems when the profile of an elastic plate part is cusped on some part of the boundary is investigated [I. N. Vekua, *Shell Theory: General Methods of Construction*, Pitman, 1985].

On the Schwarz problem for the bipotential operator

ANTONIO DI TEODORO

Universidad Simón Bolívar, Caracas, Venezuela

aditeodoro@usb.ve

We study the Schwarz problem for biharmonic inhomogeneous equations and some mixed boundary problems for this equation. We find integral representations of the solutions for the Schwarz-Dirichlet and Schwarz-Neumann problems, and for the last problem we obtain solubility conditions.

The Fourier transform method and the Markov power moment problem in controllability problems for the wave equation on a half-axis

LARISSA V. FARDIGOLA

Verkin Institute for Low Temperature Physics and Engineering, Kharkiv, Ukraine

fardigola@ukr.net

In this work sufficient conditions and necessary conditions for null-controllability and approximate null-controllability are obtained for the control system

$$w_{tt} = w_{xx} - q^2 w, \quad w(0, t) = u(t), \quad x > 0, \quad t \in (0, T),$$

where $q \geq 0$, $T > 0$, $w(\cdot, t) \in H_0^s$ ($t \in [0, T]$), $s \leq 0$ (H_0^s is a Sobolev space). We assume that the control u satisfies the restriction $|v(t)| \leq U$ a.e. on $(0, T)$, where $U > 0$ is given. Controls solving these problems are found explicitly using the Fourier transform method. Bang-bang controls solving the approximate null-controllability problem are found by means of the solutions of a system the Markov power moment problems.

Density problem of monodromy representation of Fuchsian systems

GRIGORY GIORGADZE

Tbilisi State University, Georgia

ggk@jinr.ru

In this paper we investigate some properties of monodromy representations of Fuchsian systems. The problem arises in the theory of quantum computation in connection to the construction a universal set of gates for a quantum computer.

Let X be an m -dimensional complex analytic manifold and suppose $D = \cup_{i=1}^n D_j$ is a divisor such that D_j are generic 1-codimensional submanifolds of X . Let $df = \omega f$ be a completely integrable Pfaffian system on X , where ω is a $d \times d$ -matrix valued holomorphic 1-form on $X \setminus D$. Complete integrability condition means that ω satisfies $d\omega - \omega \wedge \omega = 0$. The system $df = \omega f$ defines the monodromy representation $\rho : \pi_1(X \setminus D) \rightarrow GL(n, \mathcal{C})$. Our aim is to investigate the properties of the system $df = \omega f$ when the representation is a dense subset of $GL(n, \mathcal{C})$, i.e., $\overline{\rho(\pi_1(X \setminus D))} = GL(n, \mathcal{C})$.

When $\pi_1(X \setminus D)$ is not free, for example, a braid group, then the problem is difficult. M. Freedman, M. Larsen and Z. Wang constructed a Fuchsian system with a dense monodromy representation in $SU(n)$. When $\pi_1(X \setminus D)$ is a free group and $\dim X = 1$, then existence of the regular system with a dense monodromy representation follows from the positivity solution of the 21st Hilbert problem in the class of regular systems. In addition, if the system $df = \omega f$ is 2×2 , then the solution of the problem exists in the class of Fuchsian systems.

On a system of elliptic partial differential equations with order degeneration

GEORGE JAIANI

Tbilisi State University, Georgia

jaiani@viam.sci.tsu.ge

The talk deals with a system of elliptic partial differential equations with order degeneration arising in the first approximation of I. Vekua's hierarchical models. The well-posedness of the Dirichlet problem is investigated. The existence and uniqueness theorems are proved for the modified Dirichlet problem.

Non-local problem with special gluing condition for parabolic-hyperbolic equation

ERKINJON KARIMOV

Uzbek Academy of Sciences, Tashkent, Uzbekistan

erkinjon@gmail.com

In the present talk the unique solvability of a non-local problem with special gluing condition for a parabolic-hyperbolic equation in a mixed domain will be discussed. Also eigenvalues and corresponding eigenfunctions of the problem will be shown.

We consider an equation with parameter

$$0 = \begin{cases} u_t - u_{xx} - \lambda u, & t > 0, \\ u_{tt} - u_{xx} - \lambda u, & t < 0 \end{cases}$$

in a domain Ω , where λ is any real number. Ω is a simply connected domain located in the plane of the independent variables x, t and sectionally bounded at $t > 0$ by $AA_0 = \{(x, t) : x = 0, 0 < t < 1\}$, $A_0B_0 = \{(x, t) : t = 1, 0 < x < 1\}$, $B_0B = \{(x, t) : x = 1, 0 < t < 1\}$, and at $t < 0$ with characteristics $AC : x + t = 0$, $BC : x - t = 1$ of the given equation.

Theory of nonlinear semigroups in PDE with hysteresis

PETRA KORDULOVÁ

Mathematical Institute, Opava, Czech Republic

Petra.Kordulova@math.slu.cz

This talk deals with a quasilinear hyperbolic equation of first order with a hysteresis operator $v = \mathcal{F}[u]$:

$$\frac{\partial(u + v)}{\partial t} + \frac{\partial u}{\partial x} = 0.$$

Hysteresis is represented by a functional describing adsorption and desorption on the particles of the substance and it is possibly a discontinuous generalized play operator. We transform our equation with hysteresis into a system of differential inclusions containing an accretive operator. We investigate the solution based on the theory of nonlinear semigroups. We get the asymptotic behaviour of the solution.

Persistence of embedded eigenvalues

SARA MAAD

Stockholm University, Sweden

maad@math.su.se

Coauthors: G. Derks & B. Sandstede

Perturbation problems for embedded eigenvalues are challenging in general, since such eigenvalues cannot be separated from the rest of the spectrum. Classical perturbation theory shows that isolated eigenvalues persist under small perturbations. In contrast to this result, it is known that for a large class of operators the embedded eigenvalues disappear under arbitrarily small generic perturbations.

In this talk I will show how this can be understood for the example of the bilaplacian on a cylinder in \mathbb{R}^3 . We will also see that in this case the set of perturbations for which the embedded eigenvalue persists is a manifold of finite codimension.

Some problems for elliptic systems on the plane

NINO MANJAVIDZE

Georgian Technical University, Tbilisi, Georgia

ninomanjavidze@yahoo.com

Coauthors: G.Akhalaia & G. Makatsaria

The unified approach for investigation of wide class of discontinuous boundary value problems for generalized analytic vectors is considered. The method to obtain the solvability conditions and index formulae of corresponding boundary value problems on the plane is investigated. For Carleman-Vekua irregular equations with the coefficients with point wise singularities the correct statements of boundary value problems are indicated.

Solutions of the Robin problem for overdetermined inhomogeneous Cauchy-Riemann systems

ALIP MOHAMMED

York University, Toronto, Canada

alipm@mathstat.yorku.ca

Coauthor: M. W. Wong

The inhomogeneous Robin condition with general coefficient for the overdetermined inhomogeneous Cauchy-Riemann system of equations on the polydisc is studied using Fourier analysis. It is shown that this problem for the case of non holomorphic general coefficient, is actually a problem with essential singularity in the domain, but still well-posed under certain compatibility conditions. Under proper assumptions, the unique solution is given explicitly.

About one class of second order linear hyperbolic equations for which all of boundary consists of singular lines

NUSRAT RAJABOV

Tajik State National University, Dushanbe, Tajikistan

nusrat38@mail.ru

Led $D = \{ (x, y) : a < x < b, c < y < d \}$; $\Gamma_1 = \{a < x < b, y = c\}$, $\Gamma_2 = \{x = b, c < y < d\}$, $\Gamma_3 = \{a < x < b, y = d\}$, $\Gamma_4 = \{x = a, c < y < d\}$. In D we consider the hyperbolic equation

$$(x-a)(b-x)(y-c)(d-y)\frac{\partial^2 u}{\partial x \partial y} + (x-a)(b-x)\lambda\frac{\partial u}{\partial x} + (y-c)(d-y)\frac{\partial u}{\partial y} + \delta u = f(x, y), \quad (1)$$

where λ, μ, δ are given constants and $f(x, y)$ is a given function. By $C_{xy}^2(D)$, we denote the class of functions $u(x, y) \in C'(D)$ for which $u_{xy} \in C(D)$.

Theorem. Let $\lambda > 0$, $\mu > 0$, $\delta = \lambda\mu$, $f(x, y) \in C(\overline{D})$, $f(b, y) = 0$ with the asymptotic behavior $f(x, y) = o[(b-x)^{\gamma_1}]$ for $\gamma_1 > \mu/(b-a)$, $f(x, d) = 0$ with the asymptotic behavior $f(x, y) = o[(d-y)^{\gamma_2}]$ for $\gamma_2 > \lambda/(d-c)$. Then any solution of (1) in class $C_{xy}^2(D)$ is representable in the form $u(x, y) = K[\varphi_1(x), \varphi_2(x), \psi_1(y), \psi_2(y), f(x, y)]$, where $\varphi_1(x)$, $\psi_1(y)$ are arbitrary functions on Γ_1 and Γ_2 , $\varphi_2(x)$, $\psi_2(y)$ with $\varphi_1(x)$, $\psi_1(y)$ are related by

$$\psi_2(y) = \psi_1(y) + \int_a^b \left(\frac{t-a}{b-t} \right)^{\frac{\mu}{b-a}} \frac{f(t, y) dt}{(t-a)(b-t)(y-c)(d-y)}, \quad c < y < d,$$

$\varphi_2(x) - \varphi_1(x) = M_1[C_1, f(x, y)]$ when $a < x \leq x_0$, $\varphi_2(x) - \varphi_1(x) = M_2[C_2, f(x, y)]$ when $x_0 \leq x < b$,

$$C_2 = C_1 + \int_a^b \left(\frac{t-a}{b-t} \right)^{\frac{\mu}{b-a}} \frac{dt}{(t-a)(b-t)} \int_c^d \left(\frac{s-c}{d-c} \right)^{\frac{\lambda}{d-c}} \frac{f(t, s) ds}{(s-c)(d-c)},$$

C_1 is an arbitrary constant, and K, M_j ($j = 1, 2$) are given integral operators, $a < x_0 < b$.

We investigated properties of the solution. We obtained integral representations that applies to the correct formulation of the Darboux problems and their investigation.

Theory of a class of two-dimensional Volterra type integral equation with two super-singular lines

LUTFYA RAJABOVA

Tajik Technical University, Dushanbe, Tajikistan

mitas@mail.tj

Let D denote the rectangle $D = \{ (x, y) : a < x < a_0, b_0 < y < b \}$, $\Gamma_1 = \{a < x < x_0, y = b\}$, and $\Gamma_2 = \{x = a, y_0 < y < b\}$. In D we consider the integral equation

$$u(x, y) + \int_a^x \frac{A(t)u(t, y)}{(t-a)^\alpha} dt - \int_y^b \frac{B(s)u(x, s)}{(b-s)^\beta} ds + \int_a^x \frac{dt}{(t-a)^\alpha} \int_y^b \frac{C(t, s)u(t, s)}{(b-s)^\beta} ds = f(x, y), \quad (1)$$

where $\alpha, \beta = \text{constant} > 1$, $A(x), B(y), C(x, y), f(x, y)$ are given functions respectively in $\bar{\Gamma}_1, \bar{\Gamma}_2, \bar{D}$. In this paper the general solution to (1) is constructed in the case $\alpha > 1, \beta > 1$. Before (1), we investigated other cases. In these cases we proved that, for certain values of $A(a), B(b)$, the homogeneous version of (1) has an infinite number of linearly independent solutions, and for certain other values of $A(a), B(b)$, the homogeneous version of (1) has no solution except zero.

The non-homogeneous version of (1) for certain values of $A(a), B(b)$ is always solvable (and then there are some conditions on the right side), and for certain other values of $A(a), B(b)$, the non-homogeneous version of (1) has unique solution. In the case when $c(x, y) = -A(x)B(y)$, the solution of (1) is found in an obvious form.

Asymptotic study of anisotropic periodic rotating MHD system

SELMİ RIDHA

Faculte des Sciences de Gabes, Tunisia

ridha.selmi@isi.rnu.tn

Three-dimensional rotating anisotropic magneto hydrodynamical system MHD^ε with singular perturbation is investigated in the torus \mathbb{T}^3 . Existence and uniqueness results are proved in the anisotropic Sobolev space $H^{0,s}$ for $s > 1/2$. The proofs of the global, in time, existence of unique solution for small initial data use the energy method and a product lemma of J. Y. Chemin and coworkers. Local, in time, existence results, where the divergence-free condition play a crucial role, are based on an appropriate dyadic decomposition of the frequency space and a use compactness argument. Uniqueness result is based on an anisotropic product law in anisotropic Sobolev spaces due to D. Iftimie and uses Fourier analysis.

About the asymptotic behavior of the solution when the Rossby number ε goes to zero, we note that the singular perturbation makes the time derivatives of the velocity $\partial_t u^\varepsilon$ and the magnetic field $\partial_t b^\varepsilon$ not to be a priori bounded in ε . So the classical proofs based on taking the limit directly in the system no longer work. To overcome this difficulty, asymptotic behavior of the solution is studied using filtering process: a change of function is adopted to absorb the singular part and a limit system is obtained. Convergence of u^ε and b^ε to the solution of the limit system locally in time is proved. The mathematical study of the limit system proves that it is globally, in time, well posed. Using this point we deduce global wellposedness of the MHD^ε when ε is small, that is $\varepsilon \in [0, \varepsilon_0[$.

Spectral problems for the curl and Stokes operators in the ball

ROMEN S. SAKS

Russian Academy of Sciences, Ufa, Russia

saks@ic.bashedu.ru

The relations between eigenvalues and eigenfunctions of the curl operator R and the Stokes operator S (with a parameter ν) are indicated. We consider these operators in a bounded region G with a smooth boundary ∂G and the following boundary conditions: Dirichlet condition $v|_{\partial G} = 0$ for S , and the condition $\mathbf{n} \cdot \mathbf{u}|_{\partial G} = 0$ for R , where \mathbf{n} is the unit normal vector on ∂G .

To each eigenvalue $\pm\lambda$ of R corresponds the eigenvalue $\nu\lambda^2$ of S , and conversely to the eigenvalue μ of S corresponds two eigenvalues $\pm\sqrt{\mu/\nu}$ of R . Let \mathbf{u}^\pm and \mathbf{v} be the corresponding eigenfunctions: $R\mathbf{u}^\pm = \pm\lambda\mathbf{u}^\pm$ and $S\mathbf{v} = \nu\lambda^2\mathbf{v}$. For each \mathbf{u}^+ , it is possible to choose \mathbf{u}^- such that $\mathbf{v} = \mathbf{u}^+ + \mathbf{u}^-$. The multiplicity of every nonzero eigenvalue λ of R is finite, but the multiplicity of zero eigenvalue is infinite. In the case of a ball region of radius r , λ and \mathbf{u}^\pm are calculated explicitly. For example, $\lambda_{n,m} = \rho_{n,m}/r$, where $\rho_{n,m}$ are the positive zeros of the functions

$$\psi_n(z) \equiv \sqrt{\frac{\pi}{2z}} J_{n+1/2}(z) = (-z)^n \left(\frac{1}{z} \frac{d}{dz} \right)^n \left(\frac{\sin z}{z} \right), \quad n, m \in \mathbb{N}.$$

For a cubic region with periodic boundary conditions, the same results are in [R. S. Saks, *J. Math. Sci.* **136** (2006), 3794–3811].

A partial differential equation with a bounded set of solutions

ZAFAR D. USMANOV

Tajik Academy of Sciences, Dushanbe, Tajikistan

zafar-usmanov@rambler.ru

A specific partial differential equation in a bounded domain is studied. It is stated that without any boundary condition the equation admits either a finite number of solutions or has no solution in general.

A new approach to mixed boundary problems for the polyanalytic equation

CARMEN J. VANEGAS

Universidad Simón Bolívar, Caracas, Venezuela

cvanegas@usb.ve

Coauthors: A. Di Teodoro

We present particular combinations of mixed boundary value problems for the polyanalytic equation of order 3: Schwarz-(Dirichlet-Neumann), (Dirichlet-Neumann)-Schwarz, (Dirichlet-Neumann)-Dirichlet, Neumann-(Dirichlet-Neumann). We find integral representations for the solutions and get solubility conditions. This work is a new approach to study mixed boundary value problems.

On Hilbert boundary value problem for polyharmonic functions

YUFENG WANG

Wuhan University, China

wh_yfwang@163.com

In this article, we pose and solve a Hilbert boundary value problem for polyharmonic functions. Explicit expressions of solutions are obtained by reducing the problem into the corresponding problem of analytic functions. Moreover, a class of Dirichlet problems of polyharmonic functions may be regarded as its special case.

Complex method for two dimensional free boundary model of cancer growth

YONGZHI XU

University of Louisville, KY, USA

ysxu0001@louisville.edu

We consider a free boundary problem of nonlinear parabolic complex equation of second order in a simply or multiply connected region in complex plane. This problem is originally motivated by a free boundary problem model of cancer [Y. Xu, *Discrete Contin. Dyn. Syst. Ser. B* **4** (2004), 337–348]. The growth of cancer cell may be modeled by a reaction-diffusion equation with free boundary. We study the model for two spatial dimensions. We convert the free boundary problem to a sequence of mixed boundary value problems of nonlinear parabolic complex equations of second order with measurable coefficients. The mixed boundary value problem of nonlinear parabolic complex equations is studied by complex methods discussed by Guochun Wen in his earlier works.

Cauchy problems with monogenic initial functions

UĞUR YÜKSEL

Atılım University, Ankara, Turkey

uyuksel@atilim.edu.tr

Using a linear version of the abstract Cauchy-Kowalewski theorem, the present paper is aimed at solving the initial-value problem of the form

$$\frac{\partial u(t, x)}{\partial t} = \mathcal{L}u(t, x), \quad u(0, x) = u_0(x)$$

in $[0, T] \times G \subset \mathbb{R}_0^+ \times \mathbb{R}^{n+1}$, where the desired function $u(t, x)$ is Clifford-algebra-valued with real valued components and the operator \mathcal{L} transforms the set of all monogenic functions into itself.

Mellin-Barnes integrals and their application to a system of partial differential equations describing one-dimensional granular flows

OLEG ZHDANOV

Siberian State Aerospace University, Krasnoyarsk, Russia

onzhdanov@mail.ru

In a moving coordinate system with centre in the centre of gravity, motion is described by the hyperbolic system of equations

$$h_t + uh_x + u_xh = 0, \quad u_t + uu_x + \beta h_x = 0, \quad h(x, 0) = h_0(x), \quad u(x, 0) \equiv 0, \quad \beta = \text{const.},$$

where h is thickness, u the average speed of particle in the depth of stream, t time, and x space coordinate [V. Chugunov, J.M.H.T. Gray, K. Hutter, *Bull. Inst. Mech.*, no. 15, Tech. Univ. Darmstadt, 2004]. We find coordinates of points of intersection of characteristics and values of functions u, h at these points. In such a way we obtain the solution of system in the curvilinear triangle bounded by initial data interval and characteristics.

Using the conservation law, we reduce problem to an auxiliary system, and this system is reduced to Euler-Poisson equation. By a transformation of variables we obtain hypergeometric equation relative to the new function. Its solution is a series which converges in a circle, but the Mellin-Barnes integral is the analytic continuation into the complex plane.

Our formulas allow to calculate coordinates of points and values of functions u, h with any defined precision. T. Chalkin realised this algorithm as a program for x86 processor. We obtained the solution of Cauchy problem, and the Riemann problem is solved in a similar manner.

Characteristic singular integral equations with solutions having singularities of higher order on the real axis

SHOUGUO ZHONG

Wuhan University, China

zhangzx9@hotmail.com

Under the appropriate hypotheses subject to the unknown function and the free term and by means of our lemma, we prove the rationality of order at $x = \infty$ on two sides for the characteristic singular integral equations with solutions having singularities of higher order on the real axis X and transform the equations into solving equivalent Riemann boundary value problems with solutions having singularities of higher order and with additional conditions on X . The solutions and the solvable conditions for the former are obtained from the latter.

6 Dispersive Equations

Organizers

Vladimir Georgiev (*Università di Pisa, Italy*)

Michael Reissig (*Bergakademie Freiberg, Germany*)

General decay estimates of solution for degenerate or nondegenerate Kirchhoff equation with general dissipation

ABBÈS BENAÏSSA

Université Djillali Liabès, Sidi Bel Abbès, Algeria

benaisa.abbes@yahoo.com

In this paper, we consider the initial boundary value problem of the (degenerate or nondegenerate) Kirchhoff equation with a general dissipation of the form

$$u'' - M \left(\int_{\Omega} |\nabla u|^2 dx \right) \Delta u + \sigma(t)g(u') = 0 \quad \text{in } \Omega \times \mathbb{R}_+.$$

We prove general stability estimates using multiplier method and general weighted integral inequalities. Without imposing any growth condition at the origin on g and M , we show that the energy of the system is bounded above by a quantity, depending on σ and g , which tends to zero (as time goes to infinity). In the degenerate case, our stability estimate depends also on M . These estimates allows us to consider large class of functions g and M with general growth at the origin. We give many significant examples to illustrate how to derive from our general estimates the polynomial, exponential or logarithmic decay. The results of this paper improve and generalize many existing results in the literature.

On the free vibrations for an asymmetric beam equation

Q-HEUNG CHOI

Inha University, Incheon, Korea

qheung@inha.ac.kr

Coauthor: T. Jung

We consider the semi-linear beam equation where the nonlinear term is a functions with some powers, that is,

$$u_{tt} + u_{xxxx} + bu^+ = f(x, t, u),$$

where $u^+ = \max\{u, 0\}$ and $f(x, t, s)$ is a functions with some powers of s .

McKenna and Walter proved that if $3 < b < 15$, then at least two π -periodic solutions exist, one of which has a large amplitude. The existence of at least three solutions was later proved by Choi, Jung and McKenna by using a variational reduction method. Humphreys proved that there exists an $\epsilon > 0$ such that when $15 < b < 15 + \epsilon$ at least four periodic solutions exist. Choi and Jung suppose that $3 < b < 15$ and f is generated by eigenfunctions. Micheletti and Saccon applied the limit relative category to study multiple nontrivial solutions for a floating beam.

In this talk we use a variational approach and look for critical points of a suitable functional I on a Hilbert space H . Since the functional is strongly indefinite, it is convenient to use the notion of limit relative category.

A Littlewood-Paley approach to some inequalities for PDEs

DANIELE DEL SANTO

Università di Trieste, Italy

delsanto@univ.trieste.it

The Littlewood-Paley dyadic decomposition, as introduced in [J.-M. Bony, *Ann. Sci. École Norm. Sup.* **14** (1981), 209–246], was used in [F. Colombini & N. Lerner, *Duke Math. J.* **77** (1995), 657–698] to obtain some energy inequalities and consequently some results of well-posedness of the Cauchy problem for hyperbolic equations with coefficients depending also on the space variables. Similarly in [D. Del Santo & M. Prizzi, *J. Math. Pures Appl.* **84** (2005), 471–491] the same technique lead to a result of uniqueness for a backward parabolic equations, via Carleman inequalities. We present some new results in these two directions from [M. Cicognani, D. Del Santo & M. Reissig, *Ann. Univ. Ferrara Sez. VII Sci. Mat.* **52** (2006), 281–289] and [D. Del Santo, T. Kinoshita & M. Reissig, *Differential Integral Equations*].

Case study of solutions for the Camassa-Holm equation

MASSIMO FONTE

Austrian Academy of Sciences, Linz, Austria

massimo.fonte@oeaw.ac.at

Coauthor: A. Bressan

The purpose of this talk is to present global, continuous flow of solutions to the Camassa-Holm equation

$$u_t - u_{xxt} + 3uu_x = 2u_x u_{xx} + uu_{xx},$$

a nonlinear PDE related to the KdV equation.

We are concerned in both the *conservative solutions*, in the sense that the total energy remains a.e. constant in time, and the *dissipative* one, which appears when the energy, concentrated in the blow-up of the gradient, is annihilated. Both solutions have physical meanings, the first one in the shallow water wave regime and the second one for hyperelastic-rod wave.

Continuous dependence is performed by the accurate choice of a distance in the space H^1 . Distances defined in term of convex norm fit well in connection with linear problems, but occasionally fail when nonlinear features become dominant. The new approach is based on the construction of a distance, related on a *optimal transportation problem*, which provides the ideal tool to measure continuous dependence on the initial data for solutions to the Camassa-Holm equation. Using this new distance functional, we can construct arbitrary solutions as the uniform limit of multi-peakon solutions, and prove a general uniqueness result.

Charge neutralization condition: Large time asymptotics of solution to the Schrödinger-Poisson system with an external Coulomb potential

VLADIMIR GEORGIEV

Università di Pisa, Italy

georgiev@dm.unipi.it

Coauthor: G. Venkov

We consider the asymptotic behavior for large time of the solution to the Schrödinger-Poisson system

$$i\partial_t \psi + \Delta \psi = V\psi - \frac{\beta}{|x|}\psi, \quad \Delta V = -4\pi|\psi|^2$$

with an external Coulomb potential for $(t, x) \in [T, \infty) \times \mathbb{R}^3$ with initial data $\psi(T, x) = \psi_0(x)$. Assuming charge normalization $\|\psi\|_{L^2}^2 = \beta$ and using the pseudoconformal transform we prove that for T sufficiently large the interaction between the Hartree nonlinearity $V = \frac{1}{|x|} * |\psi|^2$ and the Coulomb potential improves time integrability.

On the well-posedness of second order weakly hyperbolic Cauchy problems under the influences of the regularity of the coefficients

FUMIHIKO HIROSAWA

Nippon Institute of Technology, Saitama, Japan

hirosawa@nit.ac.jp

We consider the loss of regularity of the solution to the Cauchy problem of second order weakly hyperbolic equations with time depending coefficients. Generally, the solution loses its regularity at the contact points of the characteristic roots, and the order of regularity loss is described by the order of contact and the order of first order derivative of the coefficients. The main purpose of our talk is that the following additional properties of the coefficients: the C^m property and stabilization of the amplitude of the characteristic roots described by an integral, which will be called *the stabilization property*, are essential for precise estimate of the loss of regularity of the solution. Thus, the order of regularity loss is described by some interactions of the following four properties of the coefficients: *contact order of characteristic roots*, *order of differentiability*, *higher order of the derivatives* and *the stabilization property*.

A unified treatment of models of thermoelasticity: Decay and diffusion phenomena

KEY JACHMANN

Bergakademie Freiberg, Germany

jachmann@math.tu-freiberg.de

We are interested in studying models of thermoelasticity from a unified point of view, i.e. we are studying Cauchy problems for linear second-order systems of the form

$$U_t + A_0 U + A_1 U_x - A_2 U_{xx} = 0$$

for d -dimensional unknowns $U = U(t, x)$ with $(t, x) \in \mathbb{R}_{>0} \times \mathbb{R}$ and A_i being real and constant $d \times d$ matrices. Using a diagonalization procedure in phase space, we are able to obtain (under certain conditions) solution representations and to apply these to the derivation of well-posedness results, L^p - L^q decay estimates and diffusion phenomena.

The above results have quite a lot of applications. In particular, they can be applied to models of thermoelasticity, i.e. to the classical ones, but also to type-2, type-3 and second sound models in one and three space dimensions, with and without lower order terms.

An application of limit relative category to a nonlinear hamiltonian system

TACKSUN JUNG

Kunsan National University, Korea

tsjung@kunsan.ac.kr

Coauthor: Q. Choi

We investigate the multiplicity of periodic solutions of the nonlinear Hamiltonian system with polynomial increase $\dot{z} = J(H_z(z))$. We look for a weak solutions $z = (p, q)$ in the Hilbert space E of the nonlinear Hamiltonian system. To find that solution we use the critical point theory induced from the relative category of the torus with two holes and the finite dimensional reduction method.

Long-term behavior of solutions of the strongly damped Boussinesq equation

VARGA KALANTAROV

Koç University, İstanbul, Turkey

vkalantarov@ku.edu.tr

The problems of global existence and non-existence of solutions to initial boundary value problems for multidimensional strongly damped Boussinesq equation will be discussed.

C^∞ -wellposedness of the Cauchy problem for 2 by 2 strictly hyperbolic systems with non-Lipschitz coefficients

TAMOTU KINOSHITA

University of Tsukuba, Japan

kinosita@math.tsukuba.ac.jp

Let us consider the Cauchy problem

$$\begin{cases} \partial_t U = A(t) \partial_x U, \\ U(0, x) = U_0(x) \end{cases} \quad \text{with} \quad A(t) = \begin{pmatrix} a(t) & b(t) \\ c(t) & d(t) \end{pmatrix} \quad (1)$$

on $[0, T] \times \mathbf{R}_x^1$. We are concerned with 2 by 2 strictly hyperbolic systems. Therefore we shall assume that $\Delta(t) = \{a(t) - d(t)\}^2 + 4b(t)c(t) \geq \exists \delta > 0$ for $t \in [0, T]$.

Theorem. Assume also $A(t) \in C^2(0, T]$. If there exists $C > 0$ such that

$$|A'(t) - \text{Tr} A'(t)|^2 + |A''(t) - \text{Tr} A''(t)| \leq \frac{C(\log t)^2}{t^2} \quad \text{for } t \in (0, T],$$

and

$$\left| \int_t^T \frac{b-c}{\sqrt{\Delta}} \left(\tan^{-1} \frac{b+c}{a-d} \right)' ds \right| \leq C |\log t| \quad \text{for } t \in (0, T],$$

then the Cauchy problem (1) is C^∞ -wellposed.

Navier-Stokes equations in aperture domains: Global existence with bounded flux and time-periodic solutions

PAOLO MAREMONTI

Seconda Università degli Studi di Napoli, Caserta, Italy

paolo.maremonti@unina2.it

In paper [F. Crispo & P. Maremonti, *Math. Methods Appl. Sci.*], we study the initial boundary value problem for the Navier-Stokes equations in a three-dimensional aperture domain Ω . Roughly speaking an aperture domain is a domain consisting of two half-spaces separated by an infinite surface with a hole. This kind of domain, beyond to the interest from a physical point of view for the variety of problems related to such a geometry, is particularly interesting from a mathematical point of view, because of the following characterization, given by J. Heywood [*Acta Math.* **136** (1976), 61–102]: $J^{1,2}(\Omega) = \{ \psi \in J_*^{1,2}(\Omega) : \Phi(\psi) = 0 \}$, where $J^{1,2}(\Omega)$ is the completion in $W^{1,2}(\Omega)$ of the set of all infinitely differentiable solenoidal vector valued functions with compact support, $J_*^{1,2}(\Omega)$ is the set of divergence free vector functions belonging to the completion in $W^{1,2}(\Omega)$ of the space of all infinitely differentiable vector valued functions with compact support, $\Phi(\psi)$ denotes the flux through the aperture. This characterization shows that the request of a solution belonging to $J^{1,2}(\Omega)$ has, as hidden request, a null flux through the aperture. This seems too strong a restriction from the physical point of view and leads us to extend the class of solutions from $J^{1,2}(\Omega)$ to $J_*^{1,2}(\Omega)$. However, in $J_*^{1,2}(\Omega)$ one has to impose an auxiliary condition for the well-posedness of the problem. One can prescribe either the total flux

through the aperture or the so-called pressure drop. In this paper we study the initial boundary value problem with, as extra condition, a prescribed flux $\alpha(t)$ through the aperture.

We are interested in proving a global existence theorem of regular solutions corresponding to suitable small initial data and flux and assuming that the flux *a priori* is just a smooth and bounded function on $(0, +\infty)$. As a consequence, we are able to prove that if the fluid presents a time-periodic flux with a period ω , the corresponding solution of the Navier-Stokes equations is a regular solution with the same period as the flux. Of course, in the case of the Stokes problem no requirement of smallness is made and we establish the first existence theorem of solutions uniformly bounded in time without assumptions of summability on the flux.

Dispersion for the Kirchhoff equations

TOKIO MATSUYAMA

Tokai University, Hiratsuka, Japan

tokio@keyaki.cc.u-tokai.ac.jp

Coauthor: M. Ruzhansky

In this talk we will present results on the time decay properties and dispersive estimates for strictly hyperbolic equations with homogeneous symbols and with time-dependent coefficients whose derivatives belong to $L^1(\mathbb{R})$. For this purpose, the method of asymptotic integration is developed for such equations, and representation formulae for solutions will be important. These formulae are analysed further to obtain time decay of L^p – L^q norms of propagators for the corresponding Cauchy problems. It turns out that the decay rates can be expressed in terms of certain geometric indices of the limiting equation and we carry out the thorough analysis of this relation. These formulae are then applied to the global solvability to Kirchhoff equations of higher order with small data. Moreover, we also obtain the time decay rate of the L^p – L^q estimates for nonlinear equations of these kind, so the time well-posedness of the corresponding equations with additional semilinearity can be treated by standard Strichartz estimates.

Scattering for evolution equations with time dependent small perturbations

KIYOSHI MOCHIZUKI

Chuo University, Tokyo, Japan

mochizuk@math.chuo-u.ac.jp

We consider the evolution equation $i\partial_t u - L_0 u + V(t)u = 0$, $t \in \mathbf{R}$, in a Hilbert space \mathcal{H} , where $i = \sqrt{-1}$, $\partial_t = \partial/\partial t$, L_0 is a selfadjoint operator with dense domain, and $V(t)$ is a uniformly bounded operator which depends continuously on $t \in \mathbf{R}$. A scattering theory is developed for this equation under the following conditions.

1. For each $t, s \in \mathbf{R}$ the evolution operator $U(t, s)$ which maps the solutions at time s to those at time t determines a bijection on \mathcal{H} .

2. The inequality $|(V(t)u, v)| \leq \eta(t)\|u\|\|v\| + \|A(t)u\|\|B(t)v\|$ holds for $u, v \in \mathcal{H}$, where $\eta(t)$ is a nonnegative L^1 function of $t \in \mathbf{R}$ and $A(t)$, $B(t)$ satisfy

$$\left| \int_0^{\pm\infty} \|B(t)e^{-itL_0} f\|^3 dt \right| \leq C_B \|f\|^2, \quad f \in \mathcal{H}, \quad \left| \int_0^{\pm\infty} \|A(t)e^{-itL_0} g\|^3 dt \right| \leq C_A \|g\|^2, \quad g \in \mathcal{H},$$

for some positive constants C_B , C_A .

Some results on spectral analysis of non-selfadjoint operators

HIDEO NAKAZAWA

Chiba Institute of Technology, Narashino, Japan

nakazawa.hideo@it-chiba.ac.jp

We treat non-selfadjoint perturbation of Schrödinger and wave equations, and explain the relation of spectral structure and asymptotic behavior of solutions. Moreover, we refer to the issue of order of pole of resolvent.

Behavior of solutions to the Cauchy problem for the damped wave equation

KENJI NISHIHARA

Waseda University, Tokyo, Japan

kenji@waseda.jp

Coauthor: T. Narazaki

We consider the Cauchy problem for the semilinear damped wave equation with exponent $\rho > 1$

$$\begin{cases} u_{tt} - \Delta u + u_t = |u|^{\rho-1}u, & (t, x) \in \mathbf{R}_+ \times \mathbf{R}^N, \\ (u, u_t)(0, x) = (u_0, u_1)(x), & x \in \mathbf{R}^N, \end{cases} \quad (DW)$$

and in relation to that for the semilinear heat equation

$$\begin{cases} \phi_t - \Delta \phi = |\phi|^{\rho-1}\phi, & (t, x) \in \mathbf{R}_+ \times \mathbf{R}^N \\ \phi(0, x) = \phi_0(x), & x \in \mathbf{R}^N. \end{cases} \quad (H)$$

It has been recognized that the solution to (DW) behaves as that to the corresponding semilinear heat equation (H) as $t \rightarrow \infty$. In fact, it holds for (H) that for small L^1 -data, a time-global solution exists if $\rho > \rho_F(N) = 1 + 2/N$, and a solution blows up within a finite time if the data is nonnegative and $\rho \leq \rho_F(N)$. Also, both the asymptotic profile in the super critical case and the blow-up time in the critical or subcritical case are obtained. For (DW) similar results follow.

In this talk we have an interest in the case that the data are not in L^1 space. We assume the data to satisfy $|\phi_0(x)| \sim C\langle x \rangle^{-kN}$, ($0 < k \leq 1$) with $\langle x \rangle = \sqrt{1 + |x|^2}$. For (H) Lee and Ni give almost complete results by the maximum principle, and the new critical exponent is $\rho_c(k) = 1 + 2/kN$. We show the global existence of the solution to (H) and its asymptotic profile in the supercritical case, without using the maximum principle, so that the method can be applicable to (DW).

Existence and blow up of solution of Cauchy problem for a multidimensional nonlinear hyperbolic equation

NECAT POLAT

Dicle University, Diyarbakır, Turkey

npolat@dicle.edu.tr

We consider the existence, both locally and globally in time, and the blow up of solutions of a multidimensional nonlinear hyperbolic equation with dispersive and damping terms.

Generalized energy conservation

MICHAEL REISSIG

Bergakademie Freiberg, Germany

reissig@math.tu-freiberg.de

We are interested in wave models possessing the property of *generalized energy conservation*. For this reason let us consider the Cauchy problem

$$\begin{cases} \partial_t^2 u - c(t)\Delta u = 0, & \text{in } [0, \infty) \times \mathbb{R}^n, \\ u(0, x) = \varphi(x), \partial_t u(0, x) = \psi(x), & \varphi, \psi \in C_0^\infty(\mathbb{R}^n), x \in \mathbb{R}^n. \end{cases}$$

We say that this model has the property of *generalized energy conservation* if the following inequalities are true:

$$C^{-1}E_W(u)(0) \leq E_W(u)(t) \leq CE_W(u)(0),$$

where C is a suitable positive constant independent of the data and where $E_W(u)(t)$ defines the classical wave energy consisting of the elastic and the kinetic energy. In this way a *blow-up of the energy* and a *decay of the energy* for $t \rightarrow \infty$ are excluded.

We discuss several conditions to the coefficients (local and non-local ones) which guarantee this property for the solutions. Generalizations to Klein-Gordon models are introduced.

Comparison principles for evolution equations

MICHAEL RUZHANSKY

Imperial College, London, UK

m.ruzhansky@imperial.ac.uk

Coauthor: M. Sugimoto

In this talk we present new comparison principles for evolution equations. With this tool, we can compare smoothing and Strichartz estimates for equations with constant or time-dependent coefficients by comparing certain quantities dependent on their symbols. As a consequence, we can show that smoothing estimates for the majority of equations are equivalent to each other.

Blowup of positive solutions of semilinear wave equations

HIROSHI UESAKA

Nihon University, Tokyo, Japan

uesaka@math.cst.nihon-u.ac.jp

We consider the Cauchy problem for a single semilinear wave equation or for a system of semilinear wave equations in \mathbf{R}^d with $d = 2, 3$. Almost all the results are shown in Caffarelli and Freidman (1986) and in Alinhac (1995). Approaches related to my research and some results for a system will be presented. In case of a single equation we treat the Cauchy problem for

$$\partial_t^2 u - \Delta u = u^p, \quad p > 1.$$

The problem has a positive local solution $u(x, t)$ which blows up in finite time T under some conditions on initial data. Blowup in a domain D means that $\lim_{t \nearrow T(x)} u(x, t) = \infty$ for each $x \in D$. We can show that $\partial_t u \geq |\nabla u|$ and that there exists a blowup boundary $t = \phi(x)$ which is Lipschitz continuous. The blowup rates of u at the blowup boundary satisfies

$$\frac{C}{(T - t)^{\frac{1}{p-1}}} \leq u(x, t) \leq \frac{C}{(T - t)^{\frac{1}{p-2}}}.$$

We can also show the same results for a system of semilinear wave equations.

On dispersive estimates for anisotropic thermo-elasticity

JENS WIRTH

University College, London, UK

j.wirth@ucl.ac.uk

Coauthor: M. Reissig

In this talk we are concerned with the Cauchy problem for the linear *thermo-elastic system*

$$\begin{aligned}U_{tt} + A(D)U + \gamma \nabla \theta &= 0 \\ \theta_t - \kappa \Delta \theta + \gamma \nabla \cdot U_t &= 0\end{aligned}$$

for the elastic displacement vector $U(t, \cdot) : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ and the thermal difference to the equilibrium state $\theta(t, \cdot) : \mathbb{R}^2 \rightarrow \mathbb{R}$. Elastic properties of the medium are described by the elasticity operator $A(D)$, which is assumed to be homogeneous of second order and positive. Typical example under consideration will be the case of *cubic media*, where

$$A(D) = -\mu \Delta - \begin{pmatrix} (\tau - \mu) \partial_1^2 & (\lambda + \mu) \partial_1 \partial_2 \\ (\lambda + \mu) \partial_1 \partial_2 & (\tau - \mu) \partial_2^2 \end{pmatrix}$$

with some structural constants $\mu, \tau \geq 0$ and $\lambda \in (-\tau - 2\mu, \tau)$. The case $\tau = \lambda + 2\mu$ corresponds to the well-known case of *isotropic media* with $A(D) = -\mu \Delta - (\lambda + \mu) \nabla^T \cdot \nabla$.

Aim of the talk is to present a new approach to handle this system and to derive L^p - L^q estimates from it. We restrict considerations to the two-dimensional case, nevertheless most of the results transfer immediately to the case of arbitrary dimensions.

Regularity and scattering for the wave equation with a critical nonlinear damping

BORISLAV YORDANOV

University of Tennessee, Knoxville, TN, USA

yordanov@math.utk.edu

Coauthors: G. Todorova & D. Ugurlu

We show that the nonlinear wave equation $u_{tt} - \Delta u + u_t^3 = 0$ is globally well-posed in radially symmetric Sobolev spaces $H_{\text{rad}}^k(\mathbf{R}^3) \times H_{\text{rad}}^{k-1}(\mathbf{R}^3)$ for all integer $k > 2$. This partially extends the well-posedness in $H^k(\mathbf{R}^3) \times H^{k-1}(\mathbf{R}^3)$ for all $k \in [1, 2]$, established by Lions and Strauss. As a consequence we obtain the global existence of C^∞ solutions with radial C_0^∞ data. The regularity problem requires smoothing and non-concentration estimates in addition to standard energy estimates, since the cubic damping is critical when $k = 2$. We also establish scattering results for initial data $(u, u_t)|_{t=0}$ in radially symmetric Sobolev spaces.

8 Inverse and Ill-Posed Problems

Organizers

Alemdar Hasanov (*Kocaeli University, İzmit, Turkey*)

Masahiro Yamamoto (*University of Tokyo, Japan*)

Sergey I. Kabanikhin (*Russian Academy of Sciences, Novosibirsk, Russia*)

Optimization-statistical method of parametrical identification and of processes control

KAMIL R. AIDA-ZADE

Azerbaijan State Oil Academy, Baku, Azerbaijan

kamil_aydazade@rambler.ru

Coauthor: M. N. Khoroshko

In this report the problems of decision-making on processes control are considered. As a rule, these problems are solved in two stages. At the first stage, the mathematical model of the process is constructed by means of parametrical identification. At this stage, the statistical data on observation of process functioning are used. At the next stage, the corresponding optimization problem is solved and optimum values of control parameters are determined.

In the present work an approach to one-stage solving problem of processes control is proposed and substantiated. This approach combines parametrical identification of the mathematical model and optimization of control parameters. The proposed approach takes into account the fact that in the case of complex, essentially nonlinear processes, it is practically impossible to solve the problem of construction of adequate mathematical model of process for the whole admissible range of parameters by regression analysis.

At the same time it is easier to solve the problem of construction of adequate mathematical model for small enough ranges of parameters, in particular, in a vicinity of any point in the area of process parameters. Therefore an iterative-type procedure for decision-making on control of object is proposed. At each iteration, optimization procedure is combined with a local refinement of parameters of the mathematical model, i.e. with solving the problem of parametrical identification, and the whole set of given statistical data is used. But all the points have weights which depend on points' distances from the current point of iterative process of optimization.

Numerical results, analysis of solving problem of the process control and comparison with results obtained when using classical two-stage method of solving problem are presented.

Robust deconvolution algorithm

ABDELHANI BOUKROUCHE

University of Guelma, Algeria

abdelhanib@yahoo.fr

Coauthors: S. Zenati

In control theory, the response of a process to an input signal is described by a convolution product of the form $y(t) = (h \star u)(t)$, where \star denotes 1D or 2D convolution, $h(t)$ is the impulse response of the degradation process or the point spread function (PSF) in image processing, and $u(t)$ is the original signal. The recorded data $y_m(t)$ is given by $y_m(t) = y(t) + v(t)$, where $v(t)$ is the noise measurement. The inverse operation consisting in the reconstruction of the signal $u(t)$ knowing $y_m(t)$ and $h(t)$, is called deconvolution.

Our contribution is the restoration of images which are degraded by blur added of noise. The given data is the recorded image and its PSF. The proposed method is a derivative of the iterative deconvolution with constraint and optimal filtering introduced by Neveux *et al.* [2000]. This method which offers the advantage of using the impulse response of the distortion process, includes a filtering step in a deconvolution algorithm with constraint. This approach

was presented by Sekko *et al.* [1997] and involves the transfer function of the distortion process from the state space model and can be considered as a modified version of classical regularization techniques. The procedure described by Sekko *et al.* [1997] can be applied to 1D as well as 2D signals. Authors have taken care to use only concepts that do not use the property of transfer causality. On the other hand this is not the case for the method given by Neveux *et al.* [2000], where the unknown signal u to be restored is considered causal. In the following it is shown that this method can be adapted to restore 1D or 2D signals without ever using the causality property. Our approach combines the method of periodicity suggested by Blanc-Feraud *et al.* [1988]. In general, the impulse response can be represented by a circulant matrix which is easily diagonalizable by a fast Fourier transform. Its diagonalization facilitates the restoration, the filtering, and decreases the number of undertaken operations as well as the size of memory space used. The proposed restoration technique is applied on a synthetic image thereby using a performance criterion it is possible to qualify the resulting enhancement.

An inverse source problem for a multi-time evolution problem

NADJIB BOUSSETILA

Université Badji Mokhtar-Annaba, Algeria

`n.boussetila@yahoo.fr`

Coauthor: F. Rebbani

Let $D = (0, T_1) \times (0, T_2)$ be a bounded rectangle in \mathbb{R}^2 with coordinates $t = (t_1, t_2) \in D$. We consider the following inverse source problem: *Find a pair $(u(t), p)$ which satisfies the differential equation*

$$\frac{\partial^2 u(t)}{\partial t_1 \partial t_2} + Au(t) = p, \quad u(t_1, 0) = 0, \quad t_1 \in [0, T_1], \quad u(0, t_2) = 0, \quad t_2 \in [0, T_2], \quad (1)$$

and the additional information

$$\Gamma(u) = u(T_1, T_2) = g, \quad (2)$$

where A is a self-adjoint positive definite linear operator with dense domain $\mathcal{D}(A)$ in the Hilbert space H and g is a prescribed function in H .

Problem (1)–(2) is not well-posed in the sense of Hadamard. Instead of the ill-posed original problem (1)–(2), we consider the well-posed problem

$$\begin{aligned} \frac{\partial^2 v(t)}{\partial t_1 \partial t_2} + A(I + \alpha A)^{-1}v(t) &= p, \quad v(t_1, 0) = 0, \quad t_1 \in [0, T_1], \quad v(0, t_2) = 0, \quad t_2 \in [0, T_2], \\ \Gamma(v) &= v(T_1, T_2) = g, \end{aligned}$$

as an approximate problem to (1)–(2). This work is mainly devoted to theoretical aspects of the method of quasi-reversibility applied to this problem. Some stability and error estimates are given under a priori regularity assumptions on the problem data.

Global in time results for some semilinear integrodifferential identification problems

FABRIZIO COLOMBO

Politecnico di Milano, Italy

`fabrizio.colombo@polimi.it`

We investigate some abstract integrodifferential inverse problems that can be applied to the heat equation with memory, to the strongly damped wave equation with memory and to some other models.

Analysis of monotonicity of input-output mappings in inverse coefficient and source problems for parabolic equations

ALİ DEMİR

Kocaeli University, İzmit, Turkey

ademir@kou.edu.tr

Inverse problems of identifying coefficient and source terms in parabolic equations from the measured output data are formulated. In the first case, the problem of determining the diffusion coefficient $k(x)$ in the parabolic equation $u_t = (k(x)u_x)_x$ with Dirichlet boundary conditions $u(0, t) = g(t)$, $u(1, t) = 0$, $0 < t < T$ from the measured flux data $f(t) := -k(0)u_x(0, t)$ is formulated. In the second case, the problem of determining source term in the parabolic equation $u_{tt} = (k(x)u_x)_x + F(x, t)$ with mixed boundary conditions $u(0, t) = 0$, $k(1)u_x(1, t) = 0$ from the measured final data $\tilde{f}(t; F) := -k(0)u_x(0, t; F)$ at the boundary $x = 0$ is formulated. For each inverse problem, structure of the input-output mappings is analyzed based on the maximum principle and the corresponding adjoint problem. Derived integral identities between the solutions of the forward problems and the corresponding adjoint problems permit one to prove the monotonicity and invertibility of the input-output mappings. Some numerical applications are presented.

An adjoint problem approach for coefficient identification in a linear hyperbolic problem

ARZU ERDEM

Kocaeli University, İzmit, Turkey

aerdem@kou.edu.tr

We consider the inverse problem of determining the unknown coefficient $k(x)$ in the hyperbolic problem

$$\begin{cases} u_{tt} = (k(x)u_x)_x, & (x, t) \in \Omega_T = \{(x, t) \in R^2 : 0 < x < 1, 0 < t \leq T\}, \\ u(x, 0) = 0, \quad u_t(x, 0) = 0, & 0 < x < 1, \\ -k(0)u_x(0, t) = \mu_0(t), \quad k(1)u_x(1, t) = \mu_1(t), & 0 < t < T, \end{cases}$$

from boundary measured data $\nu_0(t) := u(0, t)$ or/and $\nu_1(t) := u(1, t)$, $t \in (0, T]$. The inverse problem can be formulated in the operator form as

$$\Phi[k](t) = \nu_0(t), \quad \Psi[k](t) = \nu_1(t), \quad t \in (0, T].$$

The mappings $\Phi[\cdot] : \mathcal{K} \rightarrow \mathfrak{N}_0$ and $\Psi[\cdot] : \mathcal{K} \rightarrow \mathfrak{N}_1$, $\Phi[k](\cdot) := u(0, \cdot; k)$, $\Psi[k](\cdot) := u(1, \cdot; k)$, are defined to be the *input-output or coefficient-to-data mappings*. We prove monotonicity and Lipschitz continuity of these mappings. Based on these results we obtain solvability of the considered inverse problems.

On a problem of parametrical identification of a dynamical system

SAMIR Z. GULIYEV

Cybernetics Institute, Baku, Azerbaijan

kamil.aydazade@rambler.ru

A class of problems of parametrical identification of dynamic objects (in the general case, non-linear objects) with lumped parameters is studied. A special feature of the problems considered lies in the identification of averaged values of the parameters of a dynamic object, which are constant for given domains of the phase space and are optimal with respect to some performance criterion. The problem of parametrical identification is studied in the following statement. Let us consider a dynamic object described by a non-linear autonomous system of differential equations of the N th order, in which right-hand members are continuously differentiable with respect to its arguments. Suppose that the space of all possible phase states of the object is divided into a finite number of given disjoint subsets. The identifiable parameters of the object are supposed to be constant in each of these subsets. Suppose that M observations were carried out on the object when its initial states were taken from several above-mentioned subsets. As a result of these observations, the moments of the process completion, namely the moments when the system arrived at the origin of coordinates, were recorded. Then the problem of parametrical identification lies in the determination of piece-wise constant values of the parameters of the object that optimize some chosen performance criterion. The formulas for the components of the functional gradient with respect to piece-wise constant values of the identifiable parameters are obtained. These formulas allow applying efficient first order optimization numerical methods. The approach proposed can be extended to objects with distributed parameters. Numerous results of calculation experiments and their analysis are given in the work.

Special identification problem for elliptic equations

RUSLAN A. HAMIDOV

Lenkaran State University, Azerbaijan

lsu@aznet.org

Coauthor: A. D. Iskenderov

The identification of coefficients in multidimensional elliptic equations is considered, correctness of this problem is studied, existence and uniqueness theorems are proved. Let D be bounded domain in R^n with smooth boundary Γ and $x \in D$. Consider

$$A\psi + U(x)\psi = F(x), \quad x \in D, \quad (1)$$

where $\psi(x)$ is a solution, U is a control function, A is a given second order elliptic operator, and $F \in L_2(D)$ is given. Consider the boundary conditions

$$\psi|_{\Gamma} = g_0 \quad (2), \quad \frac{\partial \psi}{\partial N}|_{\Gamma} = g_1 \quad (3),$$

where $g_0 \in W_2^1(D)$ and $g_1 \in L_2(D)$ are given, and N is the conormal of Γ . Our goal is to find the unknown U and ψ from the given conditions (1)–(3). Let the set of admissible controls be $U_{ad} = \{U \in L_p(D) : U \geq 0, \|U\|_{L_p(D)} \leq d, p \geq 2\}$, where $d > 0$ is given. If $U \in U_{ad}$, a solution $\psi \in W_2^1(D)$ is understood in the generalized sense. Consider the minimization problem for the functional $J_\alpha(U) = |\psi_1(x) - \psi_2(x)|_{L_2(D)}^2 + \alpha \|U - U_0\|_{L_2(D)}^2$ in U_{ad} , where $\alpha \geq 0$ is parameter, and $U_0 \in L_p(D)$ is given. Solution of this variational problem is called the generalized solution of the identification problem (1)–(3).

Theorem. The identification problem (1)–(3) has a generalized solution.

Theorem. There exists a dense $V \subset L_p(D)$ such that for arbitrary $U_0 \in V$ and $\alpha > 0$ the identification problem (1)–(3) has a unique generalized solution.

Theorem. Let $p \leq 8/3$. Then the functional $J_\alpha(U)$ is Frechét differentiable on U_{ad} .

Identifying rock parameters of filtration model in cylindrical coordinates

ALEXANDER V. HANDZEL

North Caucasus State technical University, Stavropol, Russia

hndalxvld@yahoo.com

Coauthors: K. R. Aida-zade, L.G. Korshunova & Y. Y. Petrov

An algorithm to adapt a net model of production gas field is proposed. We consider the case of prevailing conditions when gas pool is drained mainly in dome section, the ratio of vertical permeability to horizontal one is well below 1, the form of outer gas-water contact is close to oval. For these conditions net domain in cylindrical coordinates is preferable to Descartes net approximation. In view of draining regime the flow velocities in radial direction (from periphery to dome center and vice versa) exceed angular velocities and essentially exceed vertical flows velocities. The considered conditions allow to formulate the system of discrete equations that approximate differential equation of continuous flow as a system of balance equations for net domain blocks. Only radial direction flow components are involved in these equations in implicit form. In practice initial values of rock parameters are very uncertain. The problem to refine rock parameters lies in minimization of calculated and actual pressures residuals in net model blocks. In the talk we propose an approach to solve the considered inverse problem.

Fréchet differentiability and Lipschitz continuity of cost functionals in parabolic and hyperbolic inverse problems: A unified approach

ALEMDAR HASANOV

Kocaeli University, İzmit, Turkey

ahasanov@kou.edu.tr

Inverse problems of determining source terms in parabolic, hyperbolic and beam equations from the measured data are formulated. In the first case the problem of determining the pair $w := \{F(x_1, t); T_0(t)\}$ of source terms in the parabolic equation $u_t = (k(x)u_x)_x + F(x, t)$ and Robin boundary condition $-k(l)u_x(l, t) = v[u(l, t) - T_0(t)]$ from the measured final data $\mu_T(x) = u(x, T)$ is formulated. In the second case the problems of determining the pair $w := \{F(x, t); f(t)\}$ of source terms in the hyperbolic equation $u_{tt} = (k(x)u_x)_x + F(x, t)$ and Neumann boundary condition $k(0)u_x(0, t) = f(t)$ from the measured final data $\mu(x) := u(x, T)$ or/and $\nu(x) := u_t(x, T)$ at the final moment of time $t = T$ is formulated. In the third case the inverse problems of determining the source term $F(x, t)$ in the cantilevered beam equation $u_{tt} = (EI(x)u_{xx})_{xx} + F(x, t)$ from the measured data $\mu(x) := u(x, T)$ or/and $\nu(x) := u_t(x, T)$ is considered. For all three type of inverse problems the components of the Fréchet gradient of the cost functionals, say $J_1(w) = \|u(x, T; w) - \mu(x)\|_0^2$ or/and $J_2(w) = \|u_t(x, T; w) - \nu(x)\|_0^2$ are found via the solution of corresponding adjoint problems. Lipschitz continuity of these gradients are derived. The proposed approach permit one to prove existence of a quasi-solution of the considered inverse problems, as well as to construct a monotone iteration scheme based on a gradient method.

Improving the numerical solution of ill-posed integral transforms using modified Gauss-Legendre quadrature rule

SAYED M. HASHEMIPARAST

Toosi University of Technology, Tehran, Iran

hashemiparast@yahoo.com

Numerical solution of many integral transforms using Gauss-Legendre quadrature rules mainly resulted in ill conditioned systems of nonlinear equations. In some recent works, in the numerical improvement of integrals new nodes and weights are applied to improve the solution. The main problem in these approaches is still open. In this paper we use a new modified form of Gauss-Legendre quadrature rules and determine the nodes and weights that minimize

the error of integration. This determination reduces to a system of nonlinear equations with some irregular conditions due to the Vandermonde matrices. We present an explicit solution to the fast calculation of the inversion of the matrix. Finally we compare the results with ordinary approximations and exact solutions to see the improvement.

Kantorovich theorem in Tikhonov regularization

SERGEY I. KABANIKHIN

Russian Academy of Sciences, Novosibirsk, Russia

kabanikh@math.nsc.ru

On the method of solving the Cauchy problem for the harmonic equation

TYNYSBEK S. KALMENOV

Centre of Physical and Mathematical Researches, Almaty, Kazakhstan

koshanov@list.ru

Coauthor: U. A. Iskakova

We consider the following *Cauchy problem* in a domain $\Omega = \{ (x, t) : 0 < x < \pi, -1 < t < 1 \}$: Find a solution of the Laplace equation $Lu = u_{tt} + u_{xx} = f$ in Ω satisfying the Cauchy conditions $u|_{t=-1} = \tau(x)$ and $u_t|_{t=-1} = v(x)$.

Theorem. Let $\tau(x), v(x) \in C^3[0, \pi]$, $\tau(0) = \tau(\pi) = 0$, $v(0) = v(\pi) = 0$. Then the above Cauchy problem is strongly solvable if and only if the following conditions of consistency hold: $\psi_{k,1} = \tau_{k,1} + v_{k,1} - \tilde{f}_{2k,1}$, where $\tau_{k,1}$ and $v_{k,1}$ are coefficients of the Fourier decompositions of the functions $\tau(x)$ and $(-t+1)v(x)$ on early characteristics $u_{k1}(x, t)$ of the spectral problem

$$\Delta u_{k1} = \lambda_{k1} u_{k1}(x, -t), \quad u_{k1}|_{t=-1} = 0, \quad \frac{\partial u_{k1}}{\partial t}|_{t=-1} = 0, \quad u_{k1}|_{x=0} = 0, \quad u_{k1}|_{x=\pi} = 0.$$

Here $\psi_{2k,1}$ and $\psi_{2k+1,1}$ should satisfy

$$\sum_{m=1}^{\infty} \left| \sum_{k=1}^{2m} \psi_{2k,1} c_{mk}^+ \right|^2 < \infty, \quad \sum_{m=1}^{\infty} \left| \sum_{k=1}^{2m+1} \psi_{2k+1,1} c_{mk}^- \right|^2 < \infty,$$

where c_{mk}^+ and c_{mk}^- are ortogonalization matrices depending on the variable t of the system of functions $u_{2k,1}(x, t)$ and $u_{2k+1,1}(x, t)$, respectively.

Inverse resistivity problem: Geoelectric uncertainty principle and numerical reconstruction method

BALGAISHA MUKANOVA

Al-Farabi Kazakh National University, Almaty, Kazakhstan

mbsha1@mail.ru

Coauthor: M. Orunkhanov

Mathematical model of vertical electrical sounding by using resistivity method is studied. The model leads to an inverse problem of identification of the unknown leading coefficient (conductivity) of the elliptic equation in R^2 in a slab. The direct problem is obtained in the form of mixed BVP in axisymmetric cylindrical coordinates. The additional (available measured) data is given on the upper boundary of the slab in the form of a tangential derivative. Due to ill-conditionedness of the considered inverse problem, the logarithmic transformation is applied to the unknown coefficient, and the inverse problem is studied as a minimization problem for the cost functional with respect to the reflection coefficient. The conjugate gradient (BCG) method is applied for the numerical solution of this problem. Computational experiments are performed with noise-free and noisy data.

Dynamical approach to some problems in integral geometry

BORIS PANEAH

Technion, Haifa, Israel

`peter@techunix.technion.ac.il`

The talk is mainly devoted to one of the typical problems in integral geometry: reconstructing a function f in a domain D if the integrals of f over a family of subdomains in D are known. A peculiarity of the problem in question is that we deal with bounded domains with boundaries. The solution is based on the study of the attractors of the new type noncommutative dynamical systems with *two* generators. These systems were recently introduced by the speaker in connection with the general theory of linear functional equations and boundary problems for hyperbolic differential operators. Some results can be found in speaker's papers in *Funct. Anal. Appl.*, *Trans. Amer. Math. Soc.*, and *Discrete Contin. Dyn. Syst.*

On a problem of restoring domain boundary

ANAR B. RAHIMOV

Azerbaijan State Oil Academy, Baku, Azerbaijan

`anar_r@yahoo.com`

Coauthor: V. M. Abdullayev

Numerical solution of the problem of restoring two-dimensional domain boundary of the mathematical model of a distributed system described by elliptic or parabolic equations is considered in this work. Dirichlet condition holds on the known part of the domain boundary of the problem solution and Neumann condition holds on the unknown part of the boundary. In the proposed approach, the unknown part of the boundary is represented in the form of a linear combination of basic functions with optimized unknown coefficients. The objective functional is a mean-square deviation of the values of phase state function obtained in solving the boundary problem and the values of observations at observed sources.

To numerically solve the considered problem we apply the net method and reduce the initial problem to a finite-dimensional problem of mathematical programming with constraints of a special structure. After introducing a special auxiliary impulse matrix, we make use of the specificity of the constraint conditions and obtain a system of relations for impulses. These relations constitute, in a certain sense, a conjugate system for the discrete problem, obtained as a result of approximation of the boundary problem. With the use of impulse variables, which are the solutions of the conjugate system, constructive calculation formulas for the components of the objective functional gradient have been obtained. To numerically solve the considered problem we apply first order optimization methods, in particular, gradient methods. Numerous computational experiments were carried out and the efficiency of the proposed approach to the solution of the posed inverse problems was shown.

About the control of motion of a solid body with one fixed point in a Newton field of forces

ZAURE RAKISHEVA

Al-Farabi Kazakh National University, Almaty, Kazakhstan

`zaure_ra@mail.ru`

The problem of dynamics of a solid body with one fixed point in a central Newton field of forces is researched. The first two members of the force function expansion in this case are taken into account. As is well known, the general solution of the problem cannot be reduced to quadratures. In this connection the question of researching the qualitative particularities of the motion and possibility of motion control appears.

Linear change of variables is suggested. Analysis of the characteristic roots of the transformed system of equations is carried out. For the case of instability by Lyapunov solution of the system the problem of control is stated, i.e. the small forces are searched, by addition of which the body motion is stabilized. It is shown that such forces exist and can be determined. Change of the first integrals of the controlled system is researched. With the help of reverse change of variables, nature of the controlling forces is determined and their influence upon the body motion is revealed. Thereby, for the problem of dynamics of the solid body with one fixed point in a Newton field of forces the problem of the motion control is solved.

The quasi-reversibility method for ill-posed elliptic equations

FAOUZIA REBBANI

Université Badji Mokhtar-Annaba, Algeria

rebbani@wissal.dz

Coauthors: N. Boussetila & F. Zouyed

Consider the elliptic problem

$$u''(t) - Au(t) = 0, \quad 0 < t < T, \quad u(0) = f, \quad u'(0) = 0, \quad (1)$$

where A is a self-adjoint positive definite linear operator with domain $\mathcal{D}(A)$ dense in the Hilbert space H and f is a prescribed function in H . Problem (1) (also called the Cauchy problem) is not well-posed in the sense of Hadamard. Instead of the ill-posed original problem (1) we consider the following well-posed problems:

$$v''(t) - Av(t) = 0, \quad 0 < t < T, \quad v(0) + \alpha v(T) = f, \quad v'(0) = 0,$$

$$\alpha Aw''(t) + w''(t) - Aw(t) = 0, \quad 0 < t < T, \quad w(0) = f, \quad w'(0) = 0$$

as approximations of (1). This work is mainly devoted to theoretical aspects of our method. Some stability and error estimates are given under a priori regularity assumptions on data.

Computational solution of two inverse problems for heat equation with non-local boundary conditions

BORIS RESNICK

Moscow State University, Russia

boris@resnick.ru

For the linear second-order parabolic problem

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + F(x, t), \quad 0 \leq x \leq 1, \quad 0 \leq t \leq T,$$

with non-local boundary condition $(\partial u / \partial x)(0, t) = (\partial u / \partial x)(1, t)$, we consider the following source identification inverse problem: Given $u(x, t)$ in the whole domain, determine $F(x, t)$ in that domain. We use the variational method to solve the problem reformulated as an operator equation with nonselfadjoint differential operator. Tikhonov method is used to regularize the procedure. In the second part we consider the quasilinear case

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + F(u), \quad 0 \leq x \leq 1, \quad 0 \leq t \leq T.$$

The uniqueness conditions for the forward problem are determined and the same variational approach is used to solve the inverse problem.

Network inverse problems and their applications

FIRDOUSI A. SHARIFOV

Glushkov Institute of Cybernetics, Kyiv, Ukraine

sharifov@id120.icyb.kiev.ua

Coauthor: I. V. Sergienko

The inverse problem in combinatorial optimization means an optimal modification of a cost function within given bounds such that the given feasible solution of the initial problem became an optimal one [M. Cai, X. Yang & Y. Li, *J. Optim. Theory Appl.* **104** (2000), 559–575]. Significance of the applications related to transportation and telecommunication networks of inverse problems motivated their investigations to the shortest path, minimum spanning tree, weighted bipartite matching and weighted matroid intersection problems that are a special case of finding a minimum of submodular function. An *inverse problem to the minimum cut problem* (being dual to the maximum flow problem on a network) can be formulated as follows.

With respect to given a submodular function $\omega(\cdot)$ on subsets of a finite set V , it is required to determine weights of edges in some network such that the value of the submodular function is equal to the weight of some cut in this network. As it is shown in [W. H. Cunningham, *Networks* **15** (1985), 205–215], this problem is intractable in the sense that any oracle algorithm to solve it requires exponential time. When $\omega(S)$ is a cut function, to determine nonnegative weights of edges in a polynomial time, first a network $G = (V, E)$ with the node set V and the edge set $E = \{(u, v) : u, v \in V\}$ is constructed by the method in [F. A. Sharifov *Cybernet. Systems Anal.* **37** (2001), 603–609]. Then the weights are easily computed using the functional equation in this source. It is also shown that by this method a required network can be constructed in other important problems where weights of minimum cuts can be expressed by a given parametric submodular function. Moreover, it can be used to describe the structure of large networks by analytic formulation of corresponding submodular functions.

Inverse problems for hyperbolic systems with integral conditions

YAGUB A. SHARIFOV

Cybernetics Institute, Baku, Azerbaijan

kamil.aydazade@rambler.ru

Coauthor: S. Y. Kasumov

A system of differential equations $z_{tx} = A(t, x)z + B(t, x)z_t + C(t, x)z_x + G(t, x)u$ with integral conditions $\int_0^T m(t)z(t, x)dx = \varphi(x)$ and $\int_0^T m(t)z_t(t, x)dx = \psi(x)$ is considered. An inverse problem for both boundary and optimization problems has been studied.

Classes of non-local problems are the classes of problems with non-local integral and multi-point conditions. It is significant to note that from physical considerations the conditions of this kind are quite natural and arise at mathematical modeling in cases when the characteristics of processes are unavailable or out-of-the-way for direct measuring, and therefore one has to judge them by the measurements of different observations. The similar situation is well known from the theory of inverse problems for differential equations and for optimization problems. In the theory of differential equations, inverse problems are the problems in which some coefficients of an equation or some parameters of input data along with the desired solution are subject to determination. In such problems additional conditions are given, which may be stated in the form of some average values of some physical characteristics. In the mathematical model of the studied process these conditions can be set down in the form of an integral of the desired solution. In such boundary problems, if it is possible to determine the solution then it is possible to obtain information concerning the process, which takes place on the boundary of the domain of the process flow, by means of direct calculations (i.e. as an inverse problem with unknown initial condition and integral condition of overdetermination).

Inverse problems for optimization problems of processes with non-local conditions can be stated as follows: let the measurements be taken of the trajectory values at some beforehand

unknown control at discrete points of time. It is necessary to construct an approximation for a control that has minimal norm. To solve this problem one can use the method of Tikhonov static regularization.

Stability and reconstruction for inverse corrosion problems

EVA SINCICH

Austrian Academy of Sciences, Linz, Austria

eva.sincich@oeaw.ac.at

Coauthor: G. Alessandrini

We are concerned with inverse boundary value problems arising in corrosion detection. An accurate model of corrosion requires a nonlinear relationship between the voltage and the current density on the corroded surface of the type $\partial u / \partial \nu = f(u)$. The aim of such a problem is to determine the nonlinear term f on an inaccessible portion of the boundary by performing a finite number of current and voltage measurements on the accessible one. We discuss the stability and the reconstruction issues for this inverse problem obtaining a stability result of logarithmic type and proposing a reconstruction method under some suitable a priori assumptions on the data of the problem and under mild a priori bounds on the nonlinear term itself. A simplified model of corrosion appearance reduces to the inverse problem of recovering the so-called Robin coefficient φ in a linear boundary condition $\partial u / \partial \nu = -\varphi(x)u$. We prove a Lipschitz stability estimate for φ by means of the electrostatic measurements under the further a-priori assumption of a piecewise constant Robin coefficient.

Determination of an unknown coefficient in a nonlinear elliptic problem related to the elasto-plastic torsion of a bar

SALİH TATAR

Kocaeli University, İzmit, Turkey

salih.tatar@kou.edu.tr

Coauthor: Alemdar Hasanov

The inverse problem of determining the unknown coefficient of the nonlinear differential equation of torsional creep is studied. The unknown coefficient $g = g(\xi^2)$ depends on the gradient $\xi := |\nabla u|$ of the solution $u(x)$, $x \in \Omega \subset R^n$ of the direct problem. It is proved that this gradient is bounded in C -norm. This permits one to choose the natural class of admissible coefficients for the considered inverse problem. The continuity in the norm of the Sobolev space $H^1(\Omega)$ of the solution $u(x; g)$ of the direct problem with respect to the unknown coefficient $g = g(\xi^2)$ is obtained in the following sense: $\|u(x; g) - u(x; g_m)\|_1 \rightarrow 0$ when $g_m(\eta) \rightarrow g(\eta)$ pointwise as $m \rightarrow \infty$. Based on these results the existence of a quasisolution of the inverse problem in the considered class of admissible coefficients is obtained.

Inverse problems in gene-environment networks

GERHARD-WILHELM WEBER

Middle East Technical University, Ankara, Turkey

`gweber@metu.edu.tr`

Coauthors: Z. A. Gök, B. A. Öztürk, S. Özoğur, P. Taylan, A. Tezel & Ö. Uğur

An emerging research area in computational biology and biotechnology is mathematical modeling and prediction of gene-expression patterns; it nowadays uses the theories of inverse problems and optimization to deeply understand its foundations. This talk mathematically deepens recent advances by rigorously introducing the environment and aspects of errors and uncertainty into the genetic context within the framework of matrix and interval arithmetics.

Given the data from DNA microarray experiments and environmental measurements we extract nonlinear ordinary differential equations which contain parameters that are to be determined. This is done by a generalized Chebychev approximation and generalized semi-infinite optimization. Then, time-discretized dynamical systems are studied. By a combinatorial algorithm the region of parametric stability is detected. Finally, we analyze the topological landscape of gene-environment networks in terms of structural stability and differential geometry. With aspects of spline approximation, Tikhonov regularization and with an outlook, we conclude.

Some inverse problems related to coefficient determination of a cancer growth model

YONGZHI XU

University of Louisville, KY USA

`ysxu0001@louisville.edu`

The growth of cancer cell may be modeled by a reaction-diffusion equation with free boundary. In an earlier paper, we developed a free boundary model to describe the homogeneous growth inside a cylinder, a model mimicking the growth of ductal carcinoma in situ (DCIS). Assuming that we know the coefficients of the model, we analyzed the growth tendency of DCIS. The analysis and computation of the problem show interesting results that are similar to the patterns found in DCIS [Y. Xu, *Discrete Contin. Dyn Syst. Ser. B* 4 (2004), 337–348].

In this talk we present some inverse problems related to the free boundary model of DCIS. Assuming we know the solution of the free boundary problem in a section of the cylinder, along with the known initial, boundary and free boundary conditions, we consider the inverse problem of finding the coefficients and the solution in the cylinder. The motivation of this problem is to develop mathematical methods to diagnose growth tendency of DCIS from biopsy data.

Inverse problem for a symmetrical satellite in geomagnetic field

KARLYGA ZHILISBAEVA

Al-Farabi Kazakh National University, Almaty, Kazakhstan

`zhilisbaeva@mail.ru`

Coauthor: A. Saspaeva

Mathematical model of the motion of a satellite in the geomagnetic field is proposed. The motion of the satellite around the center of mass is defined by the relationship between the magnetic moment of the satellite and the magnetic field of the earth. It is assumed that the magnetic moment of the satellite consists of a constant component and the magnetic moment of the cover. Based on the first integrals and kinematic Euler equations (or Poisson equations), the problem of construction of the dynamic Euler equations is formulated. The inverse problem consists of determination of the dynamic properties of the satellite and the magnetic field based on the given first integrals.

10 Modern Aspects of Theory of Integral Transforms and Their Applications

Organizers

Anatoly A. Kilbas (*Belarusian State University, Minsk, Belarus*)

Saburoou Saitoh (*Gunma University, Kiryu, Japan*)

Development of a model for event-oriented simulation of multiagent systems

AIGUL AKTYMBAYEVA

El-Farabi Kazakh National University, Almaty, Kazakhstan

a.aktymbaeva@kbtu.kz

Coauthor: G. Balakayeva

Components of a discrete event imitating model implementing a simulation model by using JAVA and performing an input analysis of the data and an output analysis of the simulation results are considered. Development of an imitating model of mass service system with $n \geq 1$ devices of service is constructed. On the basis of a multithreading process developed, the distributed processes are simulated with the presence of synchronization. An algorithm of event-oriented simulation is developed. The results of the functioning of a system with n devices of service are presented. The goal is to create a distributed simulation system to test various coordination mechanisms.

A change of scale transformation for unbounded functions over an analogue of conditional Wiener integrals

DONG HYUN CHO

Kyonggi University, Suwon, Korea

j94385@kyonggi.ac.kr

Let $X_\tau(x) = (x(t_1), \dots, x(t_k))$ on an analogue of Wiener space $(C[0, T], w_\varphi)$ where w_φ is a probability measure on the Borel subsets of $C[0, T]$ and $\tau : 0 < t_1 < \dots < t_k = T$ is a partition of $[0, T]$. In this talk, we establish a change of scale formula for conditional w_φ -integrals $E[G_r | X_\tau]$ of functions in $C[0, T]$ having the form

$$G_r(x) = F(x) \Psi \left(\int_0^T v_1(s) dx(s), \dots, \int_0^T v_r(s) dx(s) \right)$$

for $F \in \mathcal{S}_{w_\varphi}$ and $\Psi = \psi + \phi$ ($\psi \in L_p(\mathbb{R}^r)$, $\phi \in \hat{M}(\mathbb{R}^r)$), which need not be bounded or continuous. Here $\{v_1, \dots, v_r\}$ is an orthonormal subset of $L_2[0, T]$, \mathcal{S}_{w_φ} is a Banach algebra on $C[0, T]$ and $\hat{M}(\mathbb{R}^r)$ is the space of Fourier transforms of measures of bounded variation over \mathbb{R}^r . Finally, we show that the analytic Feynman w_φ -integral of F can be expressed as the limit of a change of scale transformation of the conditional w_φ -integral of F using an inversion formula which changes the conditional w_φ -integral of F to an ordinary w_φ -integral of F , and then we obtain a change of scale transformation for w_φ -integrals of F .

On the dilogarithm integral and convolutions

BILJANA JOLEVSKA-TUNESKA

Cyril and Methodius University, Skopje, Macedonia

`biljanaj@feit.ukim.edu.mk`

Coauthors: B. Fisher & E. Özçağ

The dilogarithm integral $\text{Li}(x)$ and its associated functions $\text{Li}_+(x)$ and $\text{Li}_-(x)$ are defined as locally summable functions on the real line. Some convolutions and neutrix convolutions of these functions and other functions are then found.

Explicit solution of ordinary and partial differential equations of fractional order by using integral transforms

ANATOLY A. KILBAS

Belarusian State University, Minsk, Belarus

`kilbas@bsu.by`

Our report is devoted to solution in closed form of ordinary and partial differential equations of fractional order by using the Laplace, Fourier and Mellin integral transforms. We give a survey of results in this field. First we present a general approach based on Laplace, Mellin and Fourier transforms to deduce solutions in closed form of non-homogeneous integral and ordinary differential equations of fractional order with constant coefficients. The equations under consideration involve the Liouville, Caputo, Hadamard and Riesz fractional integrals and derivatives, and their particular solutions are expressed in terms of convolutions involving fractional analogues of the Green function.

Next we give an application of the direct and inverse Laplace transforms to obtain general solutions of the one-dimensional homogeneous and non-homogeneous ordinary differential equations with Liouville and Caputo fractional derivatives. We also apply the one-dimensional Laplace and Mellin transforms to solve in closed form of certain classes of ordinary differential equations of fractional order with Liouville fractional derivatives and nonconstant coefficients. The results obtained are used to deduce explicit solutions of Cauchy and Cauchy-type problems for ordinary differential equations of fractional order, and, in particular, of the corresponding Cauchy problems for ordinary differential equations.

At the end of our report we deal with using multi-dimensional Laplace and Fourier transforms to solve boundary value problems for homogeneous and non-homogeneous partial differential equations of fractional order. We present the results on explicit solutions of Cauchy-type and Cauchy problems for homogeneous and inhomogeneous partial differential equations with Riemann-Liouville and Caputo partial fractional derivatives generalizing the classical heat and wave equations as well as evolution equation.

Explicit solutions of the above ordinary and partial differential equations and of the corresponding Cauchy-type and Cauchy problems are given in terms of the so-called H -function, and its special cases expressed via the Wright and generalized Wright functions; see [A. A. Kilbas & M. Saigo, *H-Transforms: Theory and Applications*, CRC, 2004], [A. Erdelyi, W. Magnus, F. Oberhettinger & F. G. Tricomi, *Higher Transcendental Functions*, vols. 1 & 3, Krieger, 1981], and A. A. Kilbas, M. Saigo & J. J. Trujillo, *Fract. Calc. Appl. Anal.* **5** (2002), 437–460].

Integral transforms with Gegenbauer and Tchebyshef functions as kernels on weighted spaces of summable functions

ANATOLY A. KILBAS

Belarusian State University, Minsk, Belarus

kilbas@bsu.by

Coauthors: H.-J. Glaeske & M. Saigo

Our report is devoted to an investigation of the integral transforms

$$\left(\mathcal{G}_{k1}^\lambda f\right)(x) = \frac{2}{\Gamma(\lambda + 1/2)} \int_x^\infty \left(1 - \frac{x^2}{t^2}\right)^{\lambda-1/2} \mathcal{G}_k^\lambda\left(\frac{x}{t}\right) f(t) \frac{dt}{t} \quad (x > 0),$$

$$\left(\mathcal{G}_{k2}^\lambda f\right)(x) = \frac{2}{\Gamma(\lambda + 1/2)} \int_0^x \left(1 - \frac{t^2}{x^2}\right)^{\lambda-1/2} \mathcal{G}_k^\lambda\left(\frac{t}{x}\right) f(t) \frac{dt}{x} \quad (x > 0),$$

and two of their modifications with complex $\lambda \in \mathbf{C}$, $\operatorname{Re}(\lambda) > -1/2$, and nonnegative integer $k \in \mathbf{N}_0 = \mathbf{N} \cup \{0\}$, $\mathbf{N} = \{1, 2, \dots\}$. Here $\mathcal{G}_k^\lambda(t) = C_k^\lambda(t)/C_k^\lambda(1)$, ($\lambda \neq 0$, $k \in \mathbf{N}_0$); $\mathcal{G}_0^0(t) = 1$, $\mathcal{G}_k^0(t) = T_k(t)$ ($k \in \mathbf{N}$), where $C_k^\lambda(t)$ is the Gegenbauer polynomial of order $k \in \mathbf{N}_0$ with index λ , and $T_k(t)$ is the Tchebyshef polynomial of first order $k \in \mathbf{N}_0$; see [A. Erdelyi, W. Magnus, F. Oberhettinger & F. G. Tricomi, *Higher Transcendental Functions*, vol. 2, Krieger, 1981]. We study properties of the above transforms in the space $\mathcal{L}_{\nu,r}$ ($\nu \in \mathbf{R}$, $1 \leq r < \infty$) of Lebesgue measurable functions f on $\mathbf{R}_+ = (0, \infty)$ such that $\int_0^\infty (|t^\nu f(t)|^r/t) dt < \infty$ ($\nu \in \mathbf{R}$, $1 \leq r < \infty$). Mapping properties such as the boundedness, the representation and the range of these transforms are proved, and their inversion formulas are established. The method of investigation is based on the representations of the above transforms as the special cases of more general integral transforms involving the so-called H -functions in the kernels; see [A. A. Kilbas & M. Saigo, *H-Transforms: Theory and Applications*, CRC, 2004].

Integration by parts formulas involving convolution products and first variations

BONG JIN KIM

Daejim University, Pacheam, Korea

bjkim98@hanmail.net

Coauthor: B. S. Kim

In this talk we establish various integration by parts formulas involving convolution products and first variations for a class of functionals defined on the space of complex-valued continuous functions on $[0, T]$ that vanish at zero. Also we obtain a formula for a functional F after it has been multiplied by n linear factors.

The derivation formula for Feynman's operational calculi

BYOUNG SOO KIM

Seoul National University of Technology, Korea

mathkbs@snut.ac.kr

Coauthor: K. S. Chang

It is important in several areas of mathematics and its applications to be able to form functions of operators. In 1951 R. Feynman invented some rules for forming functions of noncommuting operators. We introduce an approach to Feynman's operational (or functional) calculi for systems of bounded not necessarily commuting linear operators acting on a Banach space. In particular we discuss a derivation formula for Feynman's operational calculi, which is useful in studying the noncommutative differential calculus of functions of noncommuting operators.

On zero distribution of the extended Mittag-Leffler function

ANNA A. KOROLEVA

Belarus State University, Minsk, Belarus

rogosin@gmail.com

The extended Mittag-Leffler function $\mathcal{E}_{\alpha,\beta}(z)$ with $\alpha, \beta \in \mathbb{C}$ defined in Koroleva (2005) is

$$\mathcal{E}_{\alpha,\beta}(z) = \frac{1}{2\pi i} \int_{\mathcal{L}} \frac{\Gamma(s)\Gamma(1-s)}{\Gamma(\beta-\alpha s)} (-z)^{-s} ds \quad (z \neq 0).$$

Here \mathcal{L} is a specially chosen infinite contour which separates all poles of the gamma function $\Gamma(s)$ to the left, and all poles of $\Gamma(1-s)$ to the right. Our report deals with the distribution of zeros of such a function. This result generalizes asymptotic formulas for zeros of the classical Mittag-Leffler function $E_{\alpha,\beta}(z)$, $\alpha > 0$, obtained by Sedletsii (1994).

Theorem. 1. If $\Re(\alpha) > 0$, then the following asymptotic formula for zeros of $\mathcal{E}_{\alpha,\beta}(z)$ holds:

$$z_n^{\frac{1}{\alpha}} = 2\pi ni - (\tau i) \left(\log(2\pi|n|) + \frac{\pi i}{2} \operatorname{sgn}(n) \right) + \log(c) + \delta_n,$$

where $\tau = 1 + (1 - \beta)/\alpha$, $c = \alpha/\Gamma(\beta - \alpha)$, and $\delta_n = O(|n|^{-\alpha}) + O(\log(|n|)/n)$ as $n \rightarrow \pm\infty$.

2. If $\Re(\alpha) < 0$, then $\mathcal{E}_{\alpha,\beta}(z)$ is an entire function with respect to $1/z$. It has a denumerable number of zeros with the only limit point at $z = 0$. These zeros satisfy the asymptotic formula

$$z_n^{-\frac{1}{\alpha}} = 2\pi ni - (\tau i) \left(\log(2\pi|n|) + \frac{\pi i}{2} \operatorname{sgn}(n) \right) + \log(c) + \delta_n,$$

where $\tau = (\beta - 1)/\alpha$, $c = -\alpha/\Gamma(\beta - 2\alpha)$, and $\delta_n = O(|n|^\alpha) + O(\log(|n|)/n)$ as $n \rightarrow \pm\infty$.

The formulas follow from the asymptotic representation for $\mathcal{E}_{\alpha,\beta}(z)$ (Koroleva 2005).

Interpolation methods for spaces of stochastic processes

ERLAN D. NURSULTANOV

Kazakh Branch of Lomonosov State University, Astana, Kazakhstan

er-nurs@yandex.ru

Coauthor: T. U. Aubakirov

This paper introduces spaces of stochastic processes and interpolation methods for these spaces, and demonstrates the interpolation and embedding theorems. In obtaining results, Besov-type spaces with variable approximation properties are investigated.

Let a full probability space (Ω, \mathcal{F}, P) with filtering be given, i.e., $F = \{F_n\}_{n \geq 1}$ is a family of σ -algebras F_n such that $F_1 \subseteq \dots \subseteq F_n \subseteq \dots \subseteq \mathcal{F}$. Let $\{X_n\}_{n \geq 1}$ be a sequence of random variables such that X_n is measurable with respect to F_n . We call the set $X = (X_n, F_n)_{n \geq 1}$ a stochastic process. For given stochastic process $X = (X_n, F_n)_{n \geq 1}$ and $k \in \mathbb{N}$, let $\overline{X}_k = \sup\{P(A)^{-1} |\int_A X_k P(d\omega)| : A \in F_k, P(A) > 0\}$. By $N_{pq}(F)$, $0 < p, q < \infty$, we denote the ensemble of stochastic processes $X = (X_n, F_n)_{n \geq 1}$ for which $\|X\|_{N_{pq}(F)}^q = \sum_{k=1}^{\infty} k^{-1-\frac{q}{p}} \overline{X}_k^q < \infty$ and $\|X\|_{N_{p\infty}(F)} = \sup_k k^{-1/p} \overline{X}_k$. The introduction space $N_{pq}(F)$ characterizes the escalated law of large numbers for stochastic processes. So if $X \in N_{pq}(F)$, the sequence $\{X_k(\omega)/k\}_{k=1}^{\infty}$ almost surely converges to zero, and the series $\sum_{k=1}^{\infty} (k^{1-1/p} (X_k(\omega)/k))^q / k$ almost surely converges.

Let $1 < p < \infty$, $0 < q \leq \infty$, $\alpha \geq 0$, and let $X = (X_n, F_n)_{n \geq 1}$ be a martingale. We let $N_p^{\alpha q}(F) = \{X : \sum_{k=0}^{\infty} (2^{\alpha k} \overline{\Delta X}_k)^q < \infty\}$ and $N_p^{\alpha\infty}(F) = \{X : \sup_k 2^{\alpha k} \overline{\Delta X}_k < \infty\}$, where $\overline{\Delta X}_k = \sup\{P(A)^{-1+1/p} |\int_A (X_{2^k} - X_{2^{k-1}}) P(d\omega)| : A \in \mathcal{F}, P(A) > 0\}$. Suppose $X_{1/2}(\omega) \equiv 0$. The space $N_p^{\alpha q}(F)$ is a space converging martingale of the process, where parameters α, q and p characterize the velocity and metrics in which the given process converges. If $X \in N_p^{\alpha q}(F)$, then for any $A \in \mathcal{F}$ with $P(A) > 0$, the generalized conditional average $(P(A))^{-1+\frac{1}{p}} |\int_A (X_{\infty} - X_{2^k}) P(d\omega)|$ goes to zero so that $\sum_{k=0}^{\infty} (2^{\alpha k} (P(A))^{-1+1/p} |\int_A (X_{\infty} - X_{2^k}) P(d\omega)|)^q < \infty$.

For these spaces a Marcinkiewicz interpolation theorem is proved, an interpolation method is given greatly connected with characteristics of Markov's moments of the stop. The given interpolation method is used for Besov type spaces with variable approximate characteristics.

Laguerre polynomials in two variables determining special systems of partial differential equations

ZHAXYLYK N. TASMAMBETOV
Aktobe State University, Kazakhstan
 tasmam@rambler.ru

Due to the broad development of numerical methods and growth of the role of computing experiments, special functions have been an issue of growing interest. Multiple usages of the classical orthogonal polynomials of one variable is well-known. However, little is known about orthogonal polynomials of two variables, especially their relationship with partial differential equations of the second order. In works by Kroll, Sheffer, Hengelis, Suetin, equations of type such as Laguerre-Laguerre, Hermite-Hermite, Laguerre-Hermite and others are studied.

In the given work systems of Laguerre type with solutions in the form of Laguerre polynomials of two variables or a product of Laguerre polynomials of one variable are considered. Relations with previously known systems of Laguerre and other possible partial differential equations of the second order

$$\begin{aligned} xZ_{xx} + (\gamma - x)Z_x + (\beta - 1)yZ_y + nZ &= 0 \\ yZ_{yy} + (\gamma' - y)Z_y + (\beta - 1)xZ_x + mZ &= 0 \end{aligned}$$

are given, where $\gamma, \gamma', \beta, n$ and m are some constants.

Fourier-Feynman transforms on Wiener spaces

IL Yoo
Yonsei University, Wonju, Korea
 iyoo@yonsei.ac.kr

In this talk, we survey our recent results about Fourier-Feynman transforms on Wiener spaces. In particular, we introduce interesting relations among Fourier-Feynman transforms, the convolution product and the first variation of various functions on Wiener spaces.

11 Oscillation of Functional-Differential and Difference Equations

Organizers

Leonid Berezhansky (*Ben-Gurion University of the Negev, Be'er-Sheva, Israel*)

Ağacık Zafer (*Middle East Technical University, Ankara, Turkey*)

Oscillation criteria of a certain class of third order nonlinear delay differential equations

M. FAHRI AKTAŞ

Gazi University, Ankara, Turkey

mfahri@gazi.edu.tr

Coauthor: A. Tiriyaki

In this paper, using a Riccati type transformation and the integral averaging technique, some new oscillation criteria for third order delay differential equations of the form

$$(r_2(t)(r_1(t)y')')' + p(t)y' + q(t)f(y(g(t))) = 0$$

are established. The results obtained essentially improve earlier results [*J. Math. Anal. Appl.* **325** (2007), 54–68].

Oscillation criteria for a certain second order difference equation with delay and neutral terms

YAŞAR BOLAT

Kocatepe University, Afyonkarahisar, Turkey

yasarbolat@aku.edu.tr

Coauthor: Ömer Akın

In our study we are concerned with the oscillation of solutions of a general second order difference equation with delay and neutral terms whose coefficients may be positive or oscillatory sequences. We obtain some new criteria for oscillatory behaviour of the solutions of this equation. In the case of oscillating coefficients, scarce results have been obtained up to now.

Positive solutions of differential equations with time lag

JOSEF DIBLÍK

Brno University of Technology, Czech Republic

diblik@feec.vutbr.cz

Coauthor: M. Kúdelčíková

Differential equations with a time lag are considered. The existence of two asymptotically non-comparable classes of positive solutions is proved using two different techniques—the method of monotone sequences and the retract method combined with Razumikhin’s technique. With the aid of linear estimations of the right-hand side of the considered equation, inequalities for both types of positive solutions are given as well. Results are illustrated by an example. A motivation for such study came from the theory of linear differential equations with time lag. Namely, it is known that if a linear differential equation with a constant time lag and with negative coefficients admits a positive solution, then it admits *two positive and asymptotically non-comparable solutions* as well.

On the structure of solutions of linear delayed differential equations

JOSEF DIBLÍK

Brno University of Technology, Czech Republic

diblik@feec.vutbr.cz

Coauthor: M. Růžicková

In this talk we underline the importance of differential equations with a time lag for describing real phenomena. Special attention will be paid to the investigation of structure formulas of a class of linear delayed equations based on existence of a positive solution of the initial equation. We show how important an auxiliary linear equation of a special form is for obtaining structure formulae of known types of linear equations. The structure formulas for solutions of linear equations with a simple delay are derived as well.

Discreteness of the spectrum of a nonself-adjoint second order difference operator

GUSEIN S. GUSEINOV

Atılım University, Ankara, Turkey

guseinov@atilim.edu.tr

In this study, we establish conditions that ensure the discreteness of the spectrum in the Hilbert space $l^2 = \{y = (y_n)_{n=-\infty}^{\infty} : \sum_{n=-\infty}^{\infty} |y_n|^2 < \infty\}$ of the problem

$$-\Delta^2 y_{n-1} + q_n y_n = \lambda \rho_n y_n, \quad n \in \{\dots, -2, -1\} \cup \{2, 3, \dots\}, \quad y_{-1} = y_1, \quad \Delta y_{-1} = e^{2i\delta} \Delta y_1,$$

where Δ denotes the forward difference operator defined by $\Delta y_n = y_{n+1} - y_n$, $q_n \geq 0$, λ is a spectral parameter, and

$$\rho_n = \begin{cases} e^{2i\delta}, & n \leq -1, \\ e^{-2i\delta}, & n \geq 0, \end{cases}$$

for a $\delta \in [0, \frac{\pi}{2})$.

Opial inequalities on time scales and some applications

BİLLUR KAYMAKÇALAN

Georgia Southern University, Statesboro, GA, USA

billur@georgiasouthern.edu

Some unified Opial type inequalities obtained by Bohner and Kaymakçalan will be presented along with applications to various qualitative properties of several dynamic problems.

Quasi-linear functional differential equations with property A

ROMAN KOPLATADZE

Tbilisi State University, Georgia

roman@rmi.acnet.ge

Consider the differential equation

$$u^{(n)}(t) + F(u)(t) = 0, \tag{1}$$

where $n \geq 2$ and $F : C(R_+; R) \rightarrow L_{loc}(R_+; R)$ is a continuous mapping. Sufficient conditions are established for this equation to have the so-called property **A**. The results obtained are also new for the generalized Emden-Fowler type ordinary differential equation. The method by which the oscillatory properties of equation (1) are established enables one to obtain optimal conditions for (1) to have property **A** for sufficiently general equations. For some classes of

functions the sufficient conditions obtained are necessary as well. For the sake of simplicity, we give an example of the first kind equation (1) which proves the above reasoning.

Theorem. Suppose $0 < \alpha_i < \beta_i < +\infty$, $c_i \in (0, +\infty)$, $d_i \in R$ ($i = 1, \dots, m$). Then the equation

$$u^{(n)}(t) + \sum_{i=1}^m \frac{c_i}{t^{n+1}} \int_{\alpha_i t}^{\beta_i t} |u(s)|^{1+\frac{d_i}{\ln s}} \operatorname{sign} u(s) ds = 0, \quad t \geq t_0,$$

with t_0 sufficiently large, has property **A** if and only if $\max \{ \varphi(\lambda) : \lambda \in [l-1, l], l \in \{1, \dots, n-1\}, l+n \text{ is odd} \} < 1$, where $\varphi(\lambda) = \prod_{i=1}^{n-1} |\lambda - i| = (\sum_{i=1}^m c_i (\beta_i^{1+\lambda} - \alpha_i^{1+\lambda}) e^{\lambda d_i})^{-1}$.

Extensions of a theorem of Wintner on dynamic systems on time scales with asymptotically constant solutions

RAZIYE MERT

Middle East Technical University, Ankara, Turkey

raziye@metu.edu.tr

Coauthor: Ağacık Zafer

A time scale \mathbb{T} is an arbitrary nonempty closed subset of the real numbers \mathbb{R} . The most well-known examples are $\mathbb{T} = \mathbb{R}$ and $\mathbb{T} = \mathbb{Z}$. Time scale approach allows one to treat the continuous, discrete as well as more general systems simultaneously. In this study, we give a Wintner type theorem for the dynamic system $x^\Delta = A(t)x + f(t, x)$ to have an asymptotic equilibrium.

Oscillation criteria for second-order nonlinear impulsive differential equations

ABDULLAH ÖZBEKLER

Atılım University, Ankara, Turkey

abdullah@atilim.edu.tr

In this study we establish sufficient conditions for oscillation of second order nonlinear impulsive differential equations by using integral averaging technique. In particular, we extend Philos type oscillation criteria to impulsive differential equations.

Solvability and oscillation properties of functional equations relating to dynamical systems with two generators

BORIS PANEAH

Technion, Haifa, Israel

peter@technion.ac.il

The talk is devoted to the solvability of a very important class of linear functional equations of a single variable and to oscillation properties of solutions to these equations. The results to be discussed proved to be very useful when solving various problems in such diverse fields in analysis as integral and functional equations, measure theory, integral geometry, and boundary problems for hyperbolic differential equations. Some results are already published in speaker's papers in *Funct. Anal. Appl.*, *Internat. Math. Res. Notices*, and *Russ. J. Math. Phys.*

Oscillation criteria for second-order nonlinear differential equations with damping

FATOŞ TUNCAY

Eastern Mediterranean University, Gazimağusa, Turkish Republic of Northern Cyprus

fatostuncay@hotmail.com

For a class of second order nonlinear differential equations with damping, efficient oscillation theorems are derived by refining the standard integral averaging technique. In addition, interval oscillation criteria are established by using generalized Riccati transformations. They prove to be efficient in the cases where many known results fail to apply. Examples are provided to illustrate the relevance of new theorems.

Lyapunov-type inequalities for nonlinear systems

MEHMET ÜNAL

Bahçeşehir University, İstanbul, Turkey

munal@bahcesehir.edu.tr

Coauthors: A. Tiryaki & D. Çakmak

By using elementary analysis, we establish some new Lyapunov-type inequalities for nonlinear systems of differential equations whose special cases contain the well-known equations such as the Emden-Fowler type and half linear equations. The inequalities obtained here can be used as handy tools in the study of qualitative behaviour of solutions of the associated equations.

Existence of positive solutions of three-point nonlinear BVP on time scales

AHMET YANTIR

Atılım University, Ankara, Turkey

ayantir@atilim.edu.tr

Coauthor: S. G. Topal

We are concerned with proving the existence of positive solutions of second order three-point boundary value problem on time scales. The proofs are based on fixed point theorems concerning cones in a Banach space. Similar conclusions hold for m -point boundary value problems.

Interval oscillation criteria for second-order super-half-linear functional differential equations with delay and advanced arguments

AĞACIK ZAFER

Middle East Technical University, Ankara, Turkey

zafer@metu.edu.tr

We derive sufficient conditions for oscillation of second order super half-linear equations containing both delay and advanced arguments of the form

$$(\phi_\alpha(k(t)x'(t)))' + p(t)\phi_\beta(x(\tau(t))) + q(t)\phi_\gamma(x(\sigma(t))) = e(t), \quad t \geq 0,$$

where $\phi_\delta(u) = |u|^{\delta-1}u$; $\alpha > 0$, $\beta \geq \alpha$, and $\gamma \geq \alpha$ are real numbers; k, p, q, e, τ, σ are continuous real-valued functions; $\tau(t) \leq t$ and $\sigma(t) \geq t$ with $\lim_{t \rightarrow \infty} \tau(t) = \infty$. It is assumed that $p(t)$ and $q(t)$ are nonnegative on a sequence of intervals where $e(t)$ changes sign.

As an illustrative example we show that every solution of

$$(\phi_\alpha(x'(t)))' + m_1 \sin t \phi_\beta(x(t - \pi/5)) + m_2 \cos t \phi_\gamma(x(t + \pi/20)) = r_0 \cos 2t$$

is oscillatory provided that either m_1 or m_2 or r_0 is sufficiently large.

12 Pseudodifferential Operators

Organizers

Luigi Rodino (*Università di Torino, Italy*)

M. W. Wong (*York University, Toronto, Canada*)

Products of two-wavelet multipliers and their traces

VIOREL CATANĂ

Polytechnic University of Bucharest, Romania

catana.viorel@yahoo.co.uk

Following M. W. Wong's point of view in his book [*Wavelet Transforms and Localization Operators*, Birkhäuser, 2002], we give two formulas for the product of two two-wavelet multipliers $\psi T_\sigma \bar{\varphi} : L^2(\mathbb{R}^n) \rightarrow L^2(\mathbb{R}^n)$ and $\psi T_\tau \bar{\varphi} : L^2(\mathbb{R}^n) \rightarrow L^2(\mathbb{R}^n)$, where $\sigma, \tau \in L^2(\mathbb{R}^n)$ and $\varphi, \psi \in L^2(\mathbb{R}^n) \cap L^\infty(\mathbb{R}^n)$ such that $\|\varphi\|_{L^2(\mathbb{R}^n)} = \|\psi\|_{L^2(\mathbb{R}^n)} = 1$. We find also sharp estimates on the norm of two-wavelet multipliers.

Theorem 1. Then the two-wavelet multiplier $\psi T_\sigma \bar{\varphi} : L^2(\mathbb{R}^n) \rightarrow L^2(\mathbb{R}^n)$ is the same as the Weyl transform $W_{\sigma_{\varphi, \psi}} : L^2(\mathbb{R}^n) \rightarrow L^2(\mathbb{R}^n)$, where

$$\sigma_{\varphi, \psi}(x, \xi) = (2\pi)^{-n/2} \int_{\mathbb{R}^n} W(\psi, \varphi)(x, \xi - \eta) \sigma(\eta) d\eta. \quad (1)$$

Theorem 2. Then the product of two-wavelet multipliers $\psi T_\sigma \bar{\varphi} : L^2(\mathbb{R}^n) \rightarrow L^2(\mathbb{R}^n)$ and $\psi T_\tau \bar{\varphi} : L^2(\mathbb{R}^n) \rightarrow L^2(\mathbb{R}^n)$ is the same as the Weyl transform $W_\lambda : L^2(\mathbb{R}^n) \rightarrow L^2(\mathbb{R}^n)$ and λ is the function in $L^2(\mathbb{R}^n \times \mathbb{R}^n)$ given by $\hat{\lambda} = (2\pi)^{-n} (\hat{\sigma}_{\varphi, \psi} *_{1/4} \hat{\tau}_{\varphi, \psi})$, where $\sigma_{\varphi, \psi}$ and $\tau_{\varphi, \psi}$ are defined by (1).

Theorem 3. Let $\sigma \in L^1(\mathbb{R}^n)$. Then

$$(2\pi)^{-n} 2(\|\varphi\|_{L^\infty(\mathbb{R}^n)}^2 + \|\psi\|_{L^\infty(\mathbb{R}^n)}^2)^{-1} \|\tilde{\sigma}\|_{L^1(\mathbb{R}^n)} \leq \|P_{\sigma, \varphi, \psi}\|_{S_1} \leq (2\pi)^{-n} \|\sigma\|_{L^1(\mathbb{R}^n)}$$

Gevrey microlocal analysis of multi-anisotropic differential operators

RACHID CHAILI

U.S.T.O., Oran, Algeria

chaili@yahoo.fr

Coauthor: C. Bouzar

A fundamental result of Gevrey microlocal regularity due to Hörmander [*Comm. Pure Appl. Math.* **24** (1971), 671–704] is

$$WF_s(u) \subset WF_s(P(x, D)u) \cup Char(P), \quad (1)$$

where $P(x, D)$ is a differential operator with analytic coefficients in Ω . Let $WF_s(u, P(x, D))$ [see P. Bolley, J. Camus & L. Rodino, *Ren. Sem. Mat. Univ. Politec. Torino* **45** (1989), 1–61] be the Gevrey wave front of the distribution $u \in D'(\Omega)$ with respect to the iterates of the operator $P(x, D)$, then the result (1) is made more precise by the inclusion

$$WF_s(u) \subset WF_s(u, P(x, D)) \cup Char(P), \quad (2)$$

since $WF_s(u, P(x, D)) \subset WF_s(P(x, D)u)$. Various extensions and generalizations of the results (1) and (2) have been obtained, according as one considers the classes of elliptic or hypoelliptic differential operators or one considers different notions of homogeneity associated to these classes of operators, see e.g. [C. Bouzar & R. Chaili, *Arch. Math.* **76**(2001), 57–66]. The method of

Newton's polyhedron, see [S. Gindikin and L. R. Volevich, *The Method of Newton's Polyhedron in the Theory of Partial Differential Equations*, Kluwer, 1992], permits to approach differential operators with respect to their multiquasi-homogeneity.

The aim of this talk is to present a result in the spirit of [C. Bouzar & R. Chaili, *Proc. Amer. Math. Soc.* **131** (2003), 1565–1572] for a general class of multiquasi-homogeneous hypoelliptic differential operators. This analysis is done by a Gevrey inhomogeneous microlocal analysis.

On multi-anisotropic Gevrey regularity of hypoelliptic operators

AHMED DALI

Centre Universitaire de Béchar, Algeria

ahmed_dali2005@yahoo.fr

Coauthor: C. Bouzar

We show a multi-anisotropic Gevrey regularity of solutions of hypoelliptic equations. This result is an extension and precision of a classical result of Hörmander.

Weyl transforms and the heat equation for the sub-Laplacian on the Heisenberg group

APARAJITA DASGUPTA

York University, Toronto, Canada

adgupta@mathstat.yorku.ca

Coauthor: M. W. Wong

In this paper, by introducing new Sobolev spaces, we give estimates for the solution of the initial value problem for the heat equation governed by the sub-Laplacian on the Heisenberg group in terms of the initial value defined on the Heisenberg group.

Pseudodifferential operators and regularity of solutions to non linear PDE

GIANLUCA GARELLO

Università di Torino, Italy

gianluca.garello@unito.it

The pseudodifferential operators in the classical literature has been strictly related to the study of linear partial differential equations. In this short talk we expose some generalizations of the theory which allow us to obtain some results of regularity of solutions to semilinear and fully nonlinear partial differential equations.

Generalized oscillatory integrals and Fourier integral operators

CLAUDIA GARETTO

Leopold-Franzens-Universität, Innsbruck, Austria

claudia@mat1.uibk.ac.at

Coauthors: G. Hörmann & M. Oberguggenberger

We develop a theory of generalized oscillatory integrals (OIs) whose phase functions as well as amplitudes may be generalized functions of Colombeau type. Based on this, generalized Fourier integral operators (FIOs) acting on Colombeau algebras are defined. This is motivated by the need of a general framework for partial differential operators with non-smooth coefficients and distribution data. The mapping properties of these FIOs are studied, as is the microlocal Colombeau regularity for OIs and the influence of the FIO action on generalized wave front sets.

Resolvents and heat trace of cone operators

JUAN B. GIL

Pennsylvania State University, Altoona, PA, USA

jgil@psu.edu

We will review some spectral properties of elliptic cone operators, and discuss the structure and asymptotic behavior of their resolvents. For those operators whose resolvent has the proper decay, we will study the short time asymptotic expansion of the corresponding heat traces.

Hyperbolic systems of pseudodifferential operators in \mathbb{R}^n with characteristics admitting superlinear growth for $|x| \rightarrow \infty$

TODOR GRAMCHEV

Università di Cagliari, Italy

todor@unica.it

Coauthor: D. Gourdin

We investigate the Cauchy problem for first order linear hyperbolic systems of pseudodifferential equations in \mathbb{R}^n with superlinear growth of the characteristic roots for $|x| \rightarrow +\infty$. We show global well-posedness of the Cauchy problem in new classes of weighted spaces. The constructions of the spaces is determined by the type of the superlinear growth. We generalize some results of Cordes (1995), Coriasco & Rodino (1999), Ruzhansky & Sugimoto (2006). We also give examples of global representations in \mathbb{R}^n of solutions via Fourier integral operators having degenerate phase functions for which the conditions for $L^2(\mathbb{R}^n)$ estimates due to Ruzhansky & Sugimoto fail but our estimates hold.

Construction of the fundamental solution and curvature of manifolds with boundary

CHISATO IWASAKI

University of Hyogo, Himeji, Japan

iwasaki@sci.u-hyogo.ac.jp

I will talk about a method of construction of the fundamental solutions for heat equations as pseudo-differential operators with parameter the time variable, which is applicable to calculating traces of operators. By this method we obtain a generalization of a local version of the Gauss-Bonnet-Chern theorem with boundary. Our point is that we can prove the above theorems by only calculating the main term of the fundamental solution if we introduce a new weight of symbols of pseudo-differential operators.

Type 1, 1-operators defined by vanishing frequency modulation

JON JOHNSEN

Aalborg University, Denmark

jjohnsen@math.aau.dk

The talk will present a work on pseudo-differential operators of type 1, 1, in particular a general definition which is the largest one that is both compatible with negligible operators and stable under vanishing frequency modulation. (While the symbols are well known, previous works have been based on e.g. estimates in Sobolev spaces, without a general definition.) Among the results it can be mentioned that, by elaboration of the counter-examples of Ching (1972) and Hörmander (1988), type 1, 1-operators with unclosable graphs are proved to exist. Extending a work of Garello (1994), other operators are shown to lack the microlocal property as they flip the wavefront set of a nowhere differentiable function. In contrast, the definition implies the pseudo-local property for all type 1, 1-operators. The well-known fact that the support of the argument

is transported by the support of the distribution kernel is generalised to arbitrary type $1, 1$ -operators. A similar rule for spectra is also proved; for classical pseudo-differential operators this is a new result as no restrictions appear, and it gives a possibility to avoid elementary symbols. These topics will be presented as time permits.

Elliptic boundary problems on noncompact manifolds

THOMAS KRAINER

Pennsylvania State University, Altoona, PA, USA

krainer@psu.edu

We discuss Fredholm criteria and regularity results for elliptic boundary value problems on a particular class of noncompact manifolds. A simple example for our setup is Euclidean space with a noncompact obstacle removed, and the stationary Schrödinger operator with a (complex) potential and boundary conditions on the boundary of the obstacle.

More generally, the operators under consideration may be regarded as cusp operators on manifolds with corners after suitable compactification of the noncompact ends, and boundary conditions are imposed on some of the boundary hypersurfaces.

Cusp operators (with cusp degeneracy on all boundary hypersurfaces) were introduced by R. Melrose and V. Nistor in 1996 in the context of the index problem on manifolds with corners of codimension 1 (unpublished), and in the case of higher codimensions by R. Lauter and S. Moroianu (2002).

Localization operators for Stockwell transforms

YU LIU

York University, Toronto, Canada

laoniuliuyu@gmail.com

Coauthor: M. W. Wong

The resolution of the identity formula for the Stockwell transform is introduced. Localization operators corresponding to the Stockwell transforms are then defined. It is proved that under suitable conditions on the symbols, the localization operators are in descending order of complexity, paracommutators, paraproducts and Fourier multipliers.

Sampling and computations of pseudo-differential operators

ALIP MOHAMMED

York University, Toronto, Canada

alipm@mathstat.yorku.ca

Coauthor: M. W. Wong

Discrete formulas for pseudo-differential operators based on the Shannon-Whittaker formula and the Poisson summation formula are given.

Hyperbolic systems with discontinuous coefficients: Generalized wavefront sets

MICHAEL OBERGUGGENBERGER

University of Innsbruck, Austria

michael.oberguggenberger@uibk.ac.at

We study linear hyperbolic systems of pseudodifferential equations with nonsmooth, possibly discontinuous symbols and distributional data. We regularize the symbols and construct solutions in the dual of the Colombeau algebra of generalized functions. The methods are based on recent work of C. Garetto as well as energy estimates.

Solutions as elements of the Colombeau algebra itself have been known to exist since quite some time. The novelty is the introduction of the dual—this has the advantage that distributional data can be put into the equation without regularization. Thus the interplay of the singularities of the coefficients and of the data becomes more transparent.

We compute the generalized wave front set of the solution to a transport equation with discontinuous propagation speed and delta functions as initial data. The generalized wave front set turns out to have a more refined and informative structure than the wavefront set of the corresponding distributional limit.

Classes of degenerate hypoelliptic Shubin operators in Gelfand-Shilov spaces

STEVAN PILIPOVIĆ

University of Novi Sad, Serbia

pilipovic@im.ns.ac.yu

Coauthor: T. Gramchev

We propose a novel approach for the study of the uniform regularity and the decay at infinity for Shubin type pseudodifferential operators which are globally hypoelliptic but not necessarily globally and even locally elliptic. The basic idea is to use the special role of the Hermite functions for the characterization of Gelfand-Shilov spaces and to transform the problem to infinite dimensional linear systems on Banach spaces of sequences by using Fourier series expansion with respect to the Hermite functions. As applications of our general results we obtain new theorems for global hypoellipticity for classes of degenerate operators in the Shubin spaces and in inductive and projective Gelfand-Shilov spaces.

A cohomological description of the index of fibred cusp operators

FREDERIC ROCHON

Stony Brook University, NY, USA

rochon@math.sunysb.edu

Coauthor: R. Melrose

Fibred cusp operators are pseudodifferential operators defined on manifolds with boundary. Under appropriate conditions, they are Fredholm. In particular, the Atiyah-Patodi-Singer index theorem can be formulated in terms of these operators. In this talk, we will give a cohomological formulation of the index of fibred cusp operators. After introducing the appropriate cohomology theory, we will describe the Chern character from which a cohomological formula for the index can be obtained.

On local and global regularity of Fourier integral operators

MICHAEL RUZHANSKY

Imperial College, London, UK

`m.ruzhansky@imperial.ac.uk`

Coauthors: M. Sugimoto & S. Coriasco

In this talk we will discuss local and global regularity results for Fourier integral operators in different function spaces. We will present different versions of global extensions of local results, to global Sobolev spaces over L^2 and to global Sobolev spaces over L^p . We will also discuss the regularity properties of Fourier integral operators in Colombeau's spaces.

Toeplitz operators and ellipticity with global projection conditions

BERT-WOLFGANG SCHULZE

University of Potsdam, Germany

`schulze@math.uni-potsdam.de`

Ellipticity of (pseudo-)differential operators A on a compact manifold X with boundary (or with edge) Y is connected with boundary (or edge) conditions of trace and potential type, formulated in terms of global projections on Y together with an additional symbolic structure. This gives rise to operator block matrices \mathcal{A} with A in the upper left corner. We study an algebra of such operators, where ellipticity is equivalent to the Fredholm property in suitable scales of spaces: Sobolev spaces on X plus closed subspaces of Sobolev spaces on Y which are the ranges of the corresponding pseudo-differential projections. Moreover, we express parametrices of elliptic elements within our algebra and discuss spectral boundary value problems for differential operators.

Schatten-von Neumann properties of Weyl operators of Hörmander type

JOACHIM TOFT

Växjö University

`joachim.toft@vxu.se`

A fundamental result for pseudo-differential operators $\text{Op}(a)$ with symbol a reads as follows: Assume that $0 \leq \delta < \rho \leq 1$ and $r \in \mathbf{R}$. Then each $\text{Op}(a)$ with $a \in S_{\rho,\delta}^r(\mathbf{R}^{2n})$ is L^2 -continuous if and only if $S_{\rho,\delta}^r \subseteq L^\infty$ (i.e. $r \leq 0$). Here recall that $S_{\rho,\delta}^r(\mathbf{R}^{2n})$ consists of all $a \in C^\infty(\mathbf{R}^{2n})$ such that

$$|\partial_x^\alpha \partial_\xi^\beta a(x, \xi)| \leq C_{\alpha,\beta} (1 + |\xi|)^{r - \rho|\beta| + \delta|\alpha|}.$$

A somewhat weak property here is that no conclusion concerning L^2 -continuity can be done for a *particular* operator $\text{Op}(a)$ when $a \in S_{\rho,\delta}^r$ and $r > 0$.

In a recent joint paper with E. Buzano, we complete the theory at this point. More precisely, if $a \in S_{\rho,\delta}^r$, then we prove that $\text{Op}(a)$ is L^2 -continuous if and only if $a \in L^\infty$.

The theory, which contains the latter result as a special case, is formulated by means of the Hörmander-Weyl calculus, where the symbol classes $S(m, g)$ are parameterized with appropriate weight functions m and Riemannian metrics g . The continuity investigations are also performed in a broader context, which involve Schatten-von Neumann properties for such operators. Then we prove the following general result: Assume that $p \in [1, \infty]$, and the g -Planck's constant h_g satisfies $h_g^N m \in L^p$ for some $N \geq 0$. Then $\text{Op}(a)$ is a Schatten-von Neumann operator of order p if and only if $a \in L^p$. We explain these results and give some ideas of their proofs.

Pseudo-differential operators on group $SU(2)$

VILLE TURUNEN

Helsinki University of Technology, Finland

ville.turunen@hut.fi

Coauthor: M. Ruzhansky

We study pseudo-differential operators globally on matrix group $SU(2)$, without resorting to local charts. This can be done by presenting functions on the group by Fourier series obtained from the representations of the group. Due to non-commutativity, the Fourier coefficients are matrices of varying dimension. A pseudo-differential operator can be presented as a convolution operator valued mapping on the group. The corresponding Fourier coefficient matrices provide a natural global symbol of the pseudo-differential operator. Global pseudo-differential symbol inequalities and asymptotic expansions are presented in detail. As a consequence, we obtain global calculus and full symbols of pseudo-differential operators on the 3-dimensional sphere, with further geometric implications.

The zero modes and zero resonances of massless Dirac operators

TOMIO UMEDA

University of Hyogo, Himeji, Japan

umeda@sci.u-hyogo.ac.jp

Coauthor: Y. Saitō

We discuss the massless Dirac operator $H = \alpha \cdot \frac{1}{i} \nabla_x + Q(x)$, $x \in \mathbb{R}^3$, where $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ are Dirac matrices and $Q(x) = (q_{jk}(x))_{j,k}$ is a 4×4 Hermitian matrix valued function with $|q_{jk}(x)| \leq C(1 + |x|)^{-\rho}$, $\rho > 1$. (The operator H with domain $\mathcal{H}^1 = [H^1(\mathbb{R}^3)]^4$ is self-adjoint in the Hilbert space $\mathcal{L}^2 = [L^2(\mathbb{R}^3)]^4$. Here $H^1(\mathbb{R}^3)$ denotes the Sobolev space of order 1.)

By a zero mode of H , we mean an eigenfunction of the self-adjoint operator H corresponding to the eigenvalue zero. By a zero resonance, we mean a function f which satisfies $Hf = 0$ in the distributional sense and belongs to a slightly larger space than \mathcal{L}^2 , but does not belong to \mathcal{L}^2 .

It is widely recognized that zero modes and zero resonances play significant roles in various fields of mathematics and physics. However, it seems true that the properties of the zero modes and the zero resonances themselves are not well-understood. Our results are that (1) the zero mode f is a continuous function satisfying $|f(x)| \leq C(1 + |x|)^{-2}$; (2) no zero resonance exists if $\rho > 3/2$. Furthermore, we shall discuss the asymptotic property of the zero modes.

On the Cauchy problem for operators with nearly constant coefficient hyperbolic principal part

SEIICHIRO WAKABAYASHI

University of Tsukuba, Japan

wkbysh@math.tsukuba.ac.jp

We deal with hyperbolic operators whose principal symbols can be microlocally transformed to symbols depending only on the fiber variables by homogeneous canonical transformations. We call such operators “operators with nearly constant coefficient hyperbolic principal part.” Then operators with constant coefficient hyperbolic principal part and hyperbolic operators with involutive characteristics belong to this class of operators. We shall give a necessary and sufficient condition for the Cauchy problem to be C^∞ well-posed under some additional assumptions. Namely, we shall generalize “Levi condition” and prove that the generalized Levi condition is necessary and sufficient for the Cauchy problem to be C^∞ well-posed.

Absolutely referenced phase information and spectra of modified Stockwell transforms

M. W. WONG

York University, Toronto, Canada

`mwwong@mathstat.yorku.ca`

Modified Stockwell transforms are introduced to include the classical Stockwell transforms as special cases. Among them are also transforms that are reminiscent of the versatile wavelet transforms. The extension of the absolutely referenced phase information of the classical Stockwell transforms to the modified Stockwell transforms is given in terms of Riesz potentials. Based on the resolution of the identity formula, the spectrum of every modified Stockwell transform is shown to form a reproducing kernel Hilbert space.

13 Reproducing Kernels and Related Topics

Organizers

Daniel Alpay (*Ben-Gurion University of the Negev, Be'er-Sheva, Israel*)

Alain Berline (*Université des Sciences et Techniques de Languedoc, Montpellier, France*)

Saburo Saitoh (*Gunma University, Kiryu, Japan*)

Ding-Xuan Zhou (*City University of Hong Kong, Kowloon, China*)

Reproducing kernels for harmonic functions on some balls

KEIKO FUJITA

Saga University, Japan

keiko@cc.saga-u.ac.jp

We study reproducing kernels, especially Bergman and Szegő kernels, on N_p -balls defined by

$$\left\{ z : \left(\frac{(\|z\|^2 + \sqrt{\|z\|^4 - |z^2|^2})^{p/2} + (\|z\|^2 - \sqrt{\|z\|^4 - |z^2|^2})^{p/2}}{2} \right)^{1/p} < r \right\}.$$

Since the N_p -balls are domains of holomorphy, there is a holomorphic function which can not be continued analytically to the outside of the N_p -ball. However we have proved that harmonic functions on the N_p -ball with radius r can be continued harmonically to the Lie ball (N_∞ -ball) with radius $2^{1/p}r$. On the other hand, we have proved that the space of harmonic functions on the Lie ball is isomorphic to the space of holomorphic functions on the complex light cone.

Considering these facts, in this talk we will give an integral representation for harmonic functions on the N_p -ball whose integral is taken on the boundary of the complex light cone. Then we will give another proof for the harmonic continuation on the N_p -balls by using the reproducing kernel.

Numerical real inversion formulas of the Laplace transform

TSUTOMU MATSUURA

Gunma University, Kiryu, Japan

matsuura@me.gunma-u.ac.jp

We shall give a very natural and numerical real inversion formula for the Laplace transform of functions F in some natural function spaces. This integral transform is, of course, very fundamental in mathematical science. The inversion of the Laplace transform is, in general, given by a complex form, but we are interested in and are requested to obtain its real inversion in many practical problems. However, one might think that its real inversion will be essentially involved, because we must catch “analyticity” from real or discrete data. Note that the image functions of the Laplace transform are analytic on some half complex plane. In this paper, we shall give new type and very natural real inversion formulas from the viewpoints of best approximations, generalized inverses and the Tikhonov regularization by combining these fundamental ideas and methods by means of the theory of reproducing kernels. We may think that these real inversion formulas are practical and natural. We can give good error estimates in our inversion formulas. Furthermore, we shall illustrate examples by using computers.

Weighted reproducing kernels and Toeplitz operators on harmonic Bergman spaces on the real ball

RENATA OTÁHALOVÁ

Silesian University, Opava, Czech Republic

Renata.Otahalova@math.slu.cz

For the standard weighted Bergman spaces on the complex unit ball, the Berezin transform of a bounded continuous function tends to this function pointwise as the weight parameter tends to infinity. We show that this remains valid also in the context of harmonic Bergman spaces on the real unit ball of any dimension. This generalizes the recent result of C. Liu for the unit disc, as well as the original assertion concerning the holomorphic case. Along the way, we also obtain a formula for the corresponding weighted harmonic Bergman kernels.

Practical inversion formulas for linear physical systems

SABUROU SAITOH

Gunma University, Kiryu, Japan

ssaitoh@math.sci.gunma-u.ac.jp

The Tikhonov regularization is a basic important idea and method in numerical analysis. However, the extremal functions in the Tikhonov functionals are represented by using the associated singular values and singular functions for the case of compact operators. So their representations are restrictive and abstract in a sense. We shall give new representations for the case of general bounded linear operators by using the theory of reproducing kernels and introduce various concrete representations for typical cases. Our representations are both analytical and numerical. We introduce a general theory of the Tikhonov regularization using the theory of reproducing kernels containing error estimates and convergence rates. We shall present the general theory and its concrete results for the typical inverse problems for heat conduction and for the real inversion formulas with computer graphs as the evidence of the power of our inverse formulas. Furthermore, we shall refer to practical inversion formulas for physical systems.

14 Spaces of Differentiable Functions of Several Real Variables and Applications

Organizers

Viktor I. Burenkov (*Cardiff University, Wales, UK*)

Stefan G. Samko (*Universidade do Algarve, Faro, Portugal*)

Weighted Hölder estimates of singular integrals generated by a shift

SADIG K. ABDULLAYEV

Baku State University, Azerbaijan

vagif@guliyev.com

Coauthor: A. A. Akperov

Let $\mathbb{R}_{m+k,k}^+ = \{x = (x', x_{m+1}, \dots, x_{m+k}) : x_{m+1}, \dots, x_{m+k} > 0\}$, $\mathbb{S}_{m+k}^+ = \{x : |x| = 1\}$,

$$T^s u(x) = C_\nu \int_0^\pi \cdots \int_0^\pi u(x' - s', (x_{m+1}^2 - 2x_{m+1}s_{m+1} \cos \alpha_1 + s_{m+1}^2)^{1/2}, \dots, (x_{m+k}^2 - 2x_{m+k}s_{m+k} \cos \alpha_k + s_{m+k}^2)^{1/2}) \sin^{2\nu_{m+1}-1} \alpha_1 \dots \sin^{2\nu_{m+k}-1} \alpha_k d\alpha_1 \cdots d\alpha_k$$

be the generalized shift operator (GSO) generated by the Laplace-Bessel differential operator Δ_B with parameters $\nu_{m+1}, \dots, \nu_{m+k} > 0$. Consider the singular integral operator

$$Au(x) = \lim_{\varepsilon \rightarrow 0^+} \int_{\{s \in \mathbb{R}_{m+k,k}^+ : |s| > \varepsilon\}} \frac{f(\theta)}{|s|^{m+k+2\nu}} [T^s u(x)] s_{m+1}^{2\nu_{m+1}} \cdots s_{m+k}^{2\nu_{m+k}} ds.$$

generated by the GSO, where $\theta = s/|s|$, $\varepsilon > 0$, and $\nu = \nu_{m+1} + \cdots + \nu_{m+k}$. Let

$$0 < \gamma < 1, \quad 0 < \alpha - \gamma < 1, \quad 0 < \beta + \gamma < m + k. \quad (*)$$

Theorem. Suppose $f(\theta)$, $\theta \in \mathbb{S}_{m+k}^+$, is bounded, $\int_{\mathbb{S}_{m+k}^+} f(\theta) \theta_{m+1}^{2\nu_{m+1}} \cdots \theta_{m+k}^{2\nu_{m+k}} d\theta = 0$, $\exists C_f > 0$, $\forall \theta_1, \theta_2 \in \mathbb{S}_{m+k}^+$, $|f(\theta_1) - f(\theta_2)| \leq C_f |\theta_1 - \theta_2|^\delta$, $\delta > 0$. Then (1) for every $u \in H_{\alpha\beta}^\gamma(\mathbb{R}_{m+k,k})$, $Au(x)$ exists for every $x \in \mathbb{R}_{m+k,k}$, (2) if condition (*) applies and $\gamma < \delta$, then A is bounded in $H_{\alpha\beta}^\gamma(\mathbb{R}_{m+k,k})$.

On interpolation properties of generalized Besov spaces

ALEXANDRE ALMEIDA

University of Aveiro, Portugal

alexandre@mat.ua.pt

In this talk we use suitable wavelet decomposition techniques to study interpolation properties of Besov spaces with generalized smoothness.

Partial hypoellipticity of differential operators

TSEGAYE G. AYELE

Addis Ababa University, Ethiopia

tsegayeg@math.aau.edu.et

Coauthor: W. T. Bitew

In this paper we describe the condition of partial hypoellipticity of differential operators with constant coefficients in terms of fundamental solutions.

Oscillation and nonoscillation of a higher order differential equation

BAKHYT BAIDELDINOV

Kazakh Academy of Sciences, Almaty, Kazakhstan

baibak@kimep.kz

Coauthor: R. Oinarov

Let $n \geq 1$ be an integer and $\lambda > 0$ be a real number. Consider the homogeneous differential equation

$$(-1)^n (\rho(t)y^{(n)}(t))^{(n)} - \lambda \nu(t)y(t) = 0, \quad t \geq 0, \quad (1)$$

of order $2n$, where ρ and ν are continuous functions on $[0, \infty)$. A function $y(t)$, $t \geq 0$, is called a solution of equation (1) if $y(t)$ is n times continuously differentiable with the expression $\rho(t)y^{(n)}(t)$ satisfying (1) for all $t \geq 0$. Under the assumptions $\rho > 0$ and $\nu \geq 0$, sufficient conditions of conditional oscillation, strong oscillation, and nonoscillation for (1) are established. The case when ρ and ν are power functions is considered as an example.

Spectral stability of second order elliptic partial differential operators

VIKTOR I. BURENKOV

Cardiff University, Wales, UK

burenkov@cardiff.ac.uk

Coauthor: P. D. Lamberti

Recent results on the deviation of the n -th eigenvalue $\lambda_n[\Omega]$ of a non-negative self-adjoint uniformly elliptic differential operator of second order on an open set $\Omega \subset \mathbf{R}^N$ with Dirichlet or Neumann homogeneous boundary conditions upon perturbation of Ω will be presented. Two types of estimates under the appropriate assumptions on Ω_1 and Ω_2 will be under discussion:

$$|\lambda_n[\Omega_1] - \lambda_n[\Omega_2]| \leq c\lambda_n[\Omega_1]\varepsilon^\gamma, \quad 0 < \gamma \leq 1,$$

where $(\Omega_1)_\varepsilon \subset \Omega_2 \subset (\Omega_1)^\varepsilon$, $(\Omega_1)_\varepsilon = \{x \in \Omega_1 : \text{dist}(x, \partial\Omega_1) > \varepsilon\}$, $(\Omega_1)^\varepsilon = \{x \in \mathbf{R}^N : \text{dist}(x, \Omega_1) < \varepsilon\}$; and

$$|\lambda_n[\Omega_1] - \lambda_n[\Omega_2]| \leq c_n |\Omega_1 \Delta \Omega_2|,$$

where $|\Omega_1 \Delta \Omega_2|$ is the measure of the symmetric difference of Ω_1 and Ω_2 .

Some embeddings and equivalent norms of the $\mathcal{L}_{p,q}^{\lambda,s}$ spaces

DOUADI DRIHEM

M'Sila University, Algeria

douadidr@yahoo.fr

We define the space $\mathcal{L}_{p,q}^{\lambda,s}$ ($s \in \mathbb{R}$, $\lambda \geq 0$, $1 \leq p, q < \infty$) as the set of all tempered distributions f such that

$$\|f\|_{\mathcal{L}_{p,q}^{\lambda,s}} = \left(\sup_{B_J} \frac{1}{|B_J|^{\lambda/n}} \sum_{j \geq J^+} 2^{jsq} \|\mathcal{F}^{-1}(\varphi_j \mathcal{F}f)\|_{L^p(B_J)}^q \right)^{1/q} < \infty,$$

where the supremum is taken over all $J \in \mathbb{Z}$ and all ball B_J of \mathbb{R}^n with radius 2^{-J} and $(\varphi_j)_{j \in \mathbb{N}_0}$ is the smooth dyadic resolution of unity in \mathbb{R}^n . In this work we give some basic properties of this space about embeddings. Equivalent norms in terms of local means are derived. Also a characterization via approximation is proved.

Boundedness of rough B -fractional integral operators in Lorentz spaces

İSMAIL EKİNCİOĞLU

Dumlupınar University, Kütahya, Turkey

ekinci@dumlupinar.edu.tr

Coauthors: V. S. Guliyev & A. Şerbetçi

Consider the rough B -fractional integral operators

$$I_{\Omega, \alpha, \gamma} f(x) = \int_{\mathbb{R}_{k,+}^n} \frac{\Omega(y)}{|y|^{Q-\alpha}} T^y f(x) (y')^\gamma dy$$

generated by the generalized shift operator $T^y f(x) = C_{k,\gamma} \int_0^\pi \cdots \int_0^\pi f((x', y')_\alpha, x'' - y'') d\nu(\alpha)$, where $1 \leq i \leq k \leq n$, $C_{k,\gamma} = \pi^{-k/2} \prod_{i=1}^k \frac{\Gamma((\gamma_i+1)/2)}{\Gamma(\gamma_i/2)}$, $(x', y')_\alpha = ((x_1, y_1)_{\alpha_1}, \dots, (x_k, y_k)_{\alpha_k})$, $(x_i, y_i)_{\alpha_i} = (x_i^2 - 2x_i y_i \cos \alpha_i + y_i^2)^{1/2}$, $(x', x'') \in \mathbb{R}^k \times \mathbb{R}^{n-k}$, and $d\nu(\alpha) = \prod_{i=1}^k \sin^{\gamma_i-1} \alpha_i d\alpha_i$. It is well known that T^y is closely related to the Laplace-Bessel differential operator Δ_B .

Theorem. Let $1 \leq p < q < \infty$, Ω be homogeneous of degree zero on $\mathbb{R}_{k,+}^n$, and $\Omega \in L_{Q/(Q-\alpha), \gamma}(S_{k,+}^{n-1})$, $0 < \alpha < Q$.

(1) If $1 < p < Q/\alpha$ and $1 \leq r \leq s \leq \infty$, then the condition $1/p - 1/q = \alpha/Q$ is necessary and sufficient for the boundedness of $I_{\Omega, \alpha, \gamma}$ from $L_{p,r,\gamma}(\mathbb{R}_{k,+}^n)$ to $L_{q,s,\gamma}(\mathbb{R}_{k,+}^n)$.

(2) If $p = 1$ and $1 \leq r \leq \infty$, then the condition $1 - 1/q = \alpha/Q$ is necessary and sufficient for the boundedness of $I_{\Omega, \alpha, \gamma}$ from $L_{1,r,\gamma}(\mathbb{R}_{k,+}^n)$ to $L_{q,\infty,\gamma}(\mathbb{R}_{k,+}^n) \equiv WL_{q,\gamma}(\mathbb{R}_{k,+}^n)$.

The same results are given for the fractional B -maximal operator and B -Riesz potential.

Sobolev's embeddings and interpolation in Lorentz spaces

AMIRAN GOGATISHVILI

Czech Academy of Sciences, Prague, Czech Republic

gogatish@math.cas.cz

We consider Sobolev's embeddings for spaces based on rearrangement invariant spaces (not necessarily with the Fatou property) on domains with sufficiently smooth boundary in \mathbb{R}^n . We show that any optimal embedding $W^m E \subset G$, where $m < n$, can be obtained by the real interpolation of the well-known endpoint embeddings. We give also an orbital description of the optimal range space in Sobolev's embedding.

Interpolating basis in the space $C^\infty[-1, 1]$

ALEXANDER GONCHAROV

Bilkent University, Ankara, Turkey

goncha@fen.bilkent.edu.tr

We suggest a system of functionals biorthogonal to the fundamental polynomials of the first kind for the Hermite-Fejér interpolation. Under a suitable choice of zeros, the polynomials form a topological basis in the space $C^\infty[-1, 1]$. Together with the method of local interpolation, it gives a unified approach for constructing bases in spaces of C^∞ -functions on compact sets. Also we discuss the problem of growth of the sequence of Lebesgue constants corresponding to the Newton interpolation and estimate the growth of this sequence in the case of a nested family of Chebyshev's points.

$L_{p_1 r_1} \times \cdots \times L_{p_k r_k}$ boundedness for rough multilinear fractional integrals

VAGIF S. GULIYEV

Baku State University, Azerbaijan

vagif@guliyev.com

Let $\theta_j > 0$ ($j = 1, \dots, k$, $k \geq 2$) be fixed and distinct. We prove an O'Neil inequality for the k -linear convolution operators $(\mathbf{f} \otimes g)(x) = \int_{\mathbb{R}^n} f_1(x - \theta_1 y) \cdots f_k(x - \theta_k y) g(y) dy$. We define the k -sublinear fractional maximal function with a rough kernel by

$$M_{\Omega, \alpha}(\mathbf{f})(x) = \sup_{r > 0} \frac{1}{r^{n-\alpha}} \int_{|y| < r} |\Omega(y)| |f_1(x - \theta_1 y) \cdots f_k(x - \theta_k y)| dy,$$

and the k -linear fractional integral with a rough kernel by

$$I_{\Omega, \alpha}(\mathbf{f})(x) = \int_{\mathbb{R}^n} \frac{\Omega(y)}{|y|^{n-\alpha}} f_1(x - \theta_1 y) \cdots f_k(x - \theta_k y) dy,$$

where $\Omega \in L_s(S^{n-1})$, $s \geq 1$, $S^{n-1} = \{x \in \mathbb{R}^n : |x| = 1\}$, and Ω is homogeneous of degree zero on \mathbb{R}^n , i.e., $\Omega(tx) = \Omega(x)$ for all $t > 0$, $x \in \mathbb{R}^n$. We obtain rearrangement estimates for $M_{\Omega, \alpha}$ and $I_{\Omega, \alpha}$. We prove their boundedness from $L_{p_1 r_1} \times \cdots \times L_{p_k r_k}$ to L_{qr} , $1 < p < q < \infty$, $1 \leq r \leq \infty$, where p is the harmonic mean of $p_1, \dots, p_k > 1$, and r is the harmonic mean of $r_1, \dots, r_k > 1$. We show that the conditions on the parameters ensuring the boundedness can't be weakened.

Boundedness of commutators of B -Riesz potentials on $L_{p, \gamma}$ spaces

JAVANSHIR J. HASANOV

Institute of Mathematics and Mechanics, Baku, Azerbaijan

Ayhan.Serbetci@science.ankara.edu.tr

Coauthors: V. S. Guliyev & A. Şerbetçi

Consider

$$[b, I_\gamma^\alpha]f(x) = \int_{\mathbb{R}_{k,+}^n} [b(x) - b(y)] f(y) T^y |x|^{\alpha-n-|\gamma|} (y')^\gamma dy,$$

where $I_\gamma^\alpha f(x) = \int_{\mathbb{R}_{k,+}^n} f(y) T^y |x|^{\alpha-n-|\gamma|} (y')^\gamma dy$ is the B -Riesz potential and b is a locally integrable function on $\mathbb{R}_{k,+}^n$. Furthermore, let the modified B -Riesz potential be

$$\tilde{I}_\gamma^\alpha f(x) = \int_{\mathbb{R}_{k,+}^n} \left[T^y |x|^{\alpha-n-|\gamma|} - |y|^{\alpha-n-|\gamma|} \chi_{E(0,1)}(y) \right] f(y) (y')^\gamma dy.$$

The generalized shift operator is $T^y f(x) = C_{\gamma, k} \int_0^\pi \cdots \int_0^\pi f((x', y')_\beta, x'' - y'') d\nu(\beta)$, where $d\nu(\beta) = \prod_{i=1}^k \sin^{\gamma_i-1} \beta_i d\beta_i$, $x' = (x_1, \dots, x_k) \in \mathbb{R}^k$, $x'' = (x_{k+1}, \dots, x_n) \in \mathbb{R}^{n-k}$, $(x_i, y_i)_{\beta_i} = (x_i^2 - 2x_i y_i \cos \beta_i + y_i^2)^{1/2}$ for $1 \leq i \leq k$, $(x', y')_\beta = ((x_1, y_1)_{\beta_1}, \dots, (x_k, y_k)_{\beta_k})$, and $C_{\gamma, k} = \pi^{-\frac{k}{2}} \prod_{i=1}^k \frac{\Gamma((\gamma_i+1)/2)}{\Gamma(\gamma_i/2)}$.

Theorem. Let $0 < \alpha < n + |\gamma|$, $0 < \beta < 1$.

(1) For $1 < p < \frac{n+|\gamma|}{\alpha+\beta}$ and $\frac{1}{p} - \frac{1}{q} = \frac{\alpha+\beta}{n+|\gamma|}$, the commutator $[b, I_\gamma^\alpha]$ is bounded from $L_{p, \gamma}(\mathbb{R}_{k,+}^n)$ to $L_{q, \gamma}(\mathbb{R}_{k,+}^n)$ if and only if b belongs to the B -Lipschitz space $\Lambda_{\beta, \gamma}(\mathbb{R}_{k,+}^n)$.

(2) For $p = 1$ and $1 - \frac{1}{q} = \frac{\alpha+\beta}{n+|\gamma|}$, the commutator $[b, I_\gamma^\alpha]$ is bounded from $L_{1, \gamma}(\mathbb{R}_{k,+}^n)$ to $WL_{q, \gamma}(\mathbb{R}_{k,+}^n)$ if and only if $b \in \Lambda_{\beta, \gamma}(\mathbb{R}_{k,+}^n)$.

(3) For $1 < p = \frac{n+|\gamma|}{\alpha+\beta}$, $b \in \Lambda_{\beta, \gamma}(\mathbb{R}_{k,+}^n)$, the commutator $[b, \tilde{I}_\gamma^\alpha]$ is bounded from $L_{p, \gamma}(\mathbb{R}_{k,+}^n)$ to $BMO_\gamma(\mathbb{R}_{k,+}^n)$ if $b \in \Lambda_{\beta, \gamma}(\mathbb{R}_{k,+}^n)$.

Maximal operators generated by Bessel differential operators on Herz spaces

ALAKBER A. HUSEYNOV

Baku, Azerbaijan

vagif@guliyev.com

Let $\mathbb{R}_+ =]0, \infty[$, $\gamma > 0$, $B_k = \{x \in \mathbb{R}_+ : |x| < r\}$ and $A_k = B_k \setminus B_{k-1}$ for $k \in \mathbb{Z}$. Let χ_k be the characteristic function of A_k . Let $\alpha \in \mathbb{R}$, $0 < p, q \leq \infty$. Denote by $L_{p,\gamma}(\mathbb{R}_+)$ the set of measurable functions f with $\|f\|_{L_{p,\gamma}(\mathbb{R}_+)}^p = \int_{\mathbb{R}_+} |f(x)|^p x^\gamma dx < \infty$. The homogeneous Herz space is

$$K_{q,\gamma}^{\alpha,p}(\mathbb{R}_+) = \left\{ f \in L_{q,\gamma}^{loc} : \|f\|_{K_{q,\gamma}^{\alpha,p}}^p = \sum_{k=-\infty}^{\infty} 2^{k\alpha p} \|f\chi_k\|_{L_{q,\gamma}}^p < \infty \right\}.$$

Let $T^\gamma f(x) = C_\gamma \int_0^\pi f(\sqrt{x^2 + y^2 - 2xy \cos \alpha}) \sin^{\gamma-1} \alpha d\alpha$ be the B -shift operator, where $C_\gamma = \pi^{-\frac{1}{2}} \Gamma(\gamma + 1/2) \Gamma^{-1}(\gamma)$. For the function $f : \mathbb{R}_+ \rightarrow \mathbb{R}$, consider the B -maximal functions

$$M_B f(x) = \sup_{\varepsilon > 0} \varepsilon^{-1-\gamma} \int_0^\varepsilon T^\gamma |f(x)| y^\gamma dy.$$

Theorem. Let $0 < p \leq \infty$, $1 < q < \infty$ and $-(1 + \gamma)/q < \alpha < (1 + \gamma)(1 - 1/q)$. Then M_B is bounded on $K_{q,\gamma}^{\alpha,p}(\mathbb{R}_+)$.

Operators of harmonic analysis in weighted spaces with nonstandard growth

VAKHTANG KOKILASHVILI

Razmadze Mathematical Institute, Tbilisi, Georgia

kokil@rmi.acnet.ge

Coauthor: S. G. Samko

In this talk we give a development of weighted estimates of various operators of harmonic analysis in Lebesgue spaces with variable exponent $p(x)$. We present a certain $p(\cdot) \rightarrow q(\cdot)$ -version of Rubio de Francia's extrapolation theorem within the frameworks of weighted spaces $L_\rho^{p(\cdot)}$ on metric measure spaces.

By means of this extrapolation theorem and known theorems on the boundedness with Muckenhoupt weights in the case of constant p , we obtain results on weighted $p(\cdot) \rightarrow q(\cdot)$ - or $p(\cdot) \rightarrow p(\cdot)$ -boundedness—in the case of variable exponent $p(x)$ —of the following operators:

(1) potential type operators; (2) Fourier multipliers (weighted Mikhlin, Hormander and Lizorkin-type theorems); (3) multipliers of trigonometric Fourier series; (3) majorants of partial sums of Fourier series; (4) singular integral operators on Carleson curves and in Euclidean setting; (5) Feffermann-Stein function; (6) some vector-valued operators.

On embeddings and multipliers of weighted Sobolev spaces

LEILI KUSSAINOVA

Kazakh Academy of Sciences, Karaganda, Kazakhstan

leili2006@mail.ru

We present some results on embeddings and multipliers of weighted Sobolev spaces with norm $\|f; W_{p,r}^l(\rho, v)\| = \|\nabla_l f\|_{p,\rho} + \|f\|_{r,v}$, $1 < p, r < \infty$, where a weighted pair (ρ, v) satisfies a local condition (II). Let v be a weight on a domain $G \subset \mathbb{R}^n$. We set $\|f\|_{p,v}^p = \int_G |f|^p v$, $v(E) = \int_E v$, $|E| = \int_E dx$. Let $Q = Q_d = Q_d(x) = \{y \in \mathbb{R}^n : |y_i - x_i| < d/2, i = 1, \dots, n\}$ ($d > 0$). Let $d(\cdot)$ be a positive function in G such that $Q(x) = Q_{d(x)}(x) \subset G$ for all $x \in G$. If there exist $0 \leq \varepsilon < 1$, $0 \leq \delta < 1$ and $\gamma > 0$ such that $d(x)^{l-n} (\tilde{\rho}(Q))^{1/p'}$ $\inf\{(v(Q \setminus e))^{1/r} : e \subset Q, |e| \leq \delta|Q|\} \geq \gamma$, a.e. in G , where $Q = Q_\varepsilon(x) = (1 - \varepsilon)Q(x)$, $\tilde{\rho} = \rho^{1-p'}$, $p' = p/(p-1)$, we write $(\rho, v) \in \Pi_{(\delta, \varepsilon), p, r, l}$ with respect to $d(\cdot)$. Assume $1 < p \leq q < \infty$.

Theorem 1. Let $0 \leq k < l$, $1 \leq r \leq q$ and $(\rho, v) \in \Pi_{(\delta, \varepsilon), p, r, l}$. If a Borel measure μ on G satisfies $K(x) = (\tilde{\rho}(Q_{(\varepsilon)}(x)))^{1/p'} \sup\{h^{l-k-n}(\mu(Q_h))^{1/q} : Q_h \subset Q_{(\varepsilon)}(x)\} \leq C < \infty$ a.e. in G , then $\|\nabla_k f\|_{q, \mu} \leq cK\|f\|_{W_{p, r}^l(\rho, v)}$ for $f \in C^\infty(G) \cap W_{p, r}^l(\rho, v)$, where $K = \text{ess sup}_{x \in G} K(x)$.

Theorem 1 contains Sobolev embedding theorems, embedding theorems of spaces $W_{p, p}^l(\rho, v)$ with power-type weights, theorems on traces.

We consider a local maximal operator $M^*f(x) = \sup_{x \in Q \in \mathcal{B}} |Q|^{-1} \int_Q |f|$ with respect to a regular $d(x)$, where $B = \bigcup_{x \in G} \{Q : Q \subset Q_{(\varepsilon)}(x)\}$. Let $W_1 = W_p^l(1, v)$, $W_2 = W_q^m(\omega_1, \omega_2)$, where $(1, v) \in \Pi_{(\delta, \varepsilon), p, p, l}$ and ω_i have regular maximal function $M^*\omega_i$. A multiplier $\gamma \in M(W_1 \rightarrow W_2)$ is a function such that $\gamma W_1 \subset W_2$. We set $K(x|\gamma, \omega) = \sup\{h^{l-n/p}(\int_{Q_h} |\nabla_m \gamma|^q M^*\omega)^{1/q} + h^{-m}(\int_{Q_h} |\gamma|^q M^*\omega)^{1/q} : Q_h \subset Q_{(\varepsilon)}(x)\}$ for integers $0 < m < l$.

Theorem 2. Assume $pl \neq n$ and $K = \text{ess sup}\{K(x|\gamma, \omega_1 + \omega_2) : x \in G\} < \infty$. Then $\gamma \in M(W_1 \rightarrow W_2)$ with norm $\|\gamma\| \leq cK$.

A significant consequence of Theorem 2 is the weak estimate

$$\|\gamma\|_{M(W_p^l(\mathbb{R}^n) \rightarrow W_p^m(\mathbb{R}^n))} \asymp \sup\{h^{l-n/p}(\|\nabla_m \gamma; L_p(Q_h)\| + h^{-m}\|\gamma; L_p(Q_h)\|) : Q_h, 0 < h \leq 1\}.$$

We prove that these results cannot be improved in $\Pi_{(\delta, \varepsilon), p, p, l}$ with respect to admissible $d(x)$.

Spectral stability of the p -Laplacian

PIER DOMENICO LAMBERTI

Università di Padova, Italy

lamberti@unipd.it

Coauthor: V. I. Burenkov

Let $p \in]1, \infty[$ and Ω be a bounded domain in \mathbb{R}^N . We consider the well-known eigenvalue problem

$$-\Delta_p u = \lambda |u|^{p-2} u,$$

for $\lambda \in \mathbb{R}$ and u in the Sobolev space $W_0^{1, p}(\Omega)$, where $\Delta_p u \equiv \text{div}(|Du|^{p-2} Du)$ is the p -Laplacian.

As in the linear case $p = 2$, the p -Laplacian has a sequence of eigenvalues $\lambda_{p, j}[\Omega]$, $j \in \mathbb{N}$, obtained by means of a Min-Max Principle. Here we consider the dependence of $\lambda_{p, j}[\Omega]$ on Ω and we present estimates for the variation of such eigenvalues upon variation of Ω . Special attention will be devoted to the first eigenvalue for which an estimate of the type

$$\lambda_{p, 1}[\Omega_1] \leq \lambda_{p, 1}[\Omega_2] \leq \lambda_{p, 1}[\Omega_1] + c|\Omega_1 \setminus \Omega_2|$$

holds whenever Ω_1 is sufficiently smooth, $\Omega_2 \subset \Omega_1$ and the Lebesgue measure $|\Omega_1 \setminus \Omega_2|$ is sufficiently small. (Here c is a positive constant independent of Ω_2 .)

On the boundedness of one-sided singular integrals in Lebesgue spaces with variable exponent

ALEXANDER MESKHI

Razmadze Mathematical Institute, Tbilisi, Georgia

meskhi@rmi.acnet.ge

Coauthors: D. E. Edmunds & V. Kokilashvili.

The boundedness of one-sided potentials and singular integrals are established. Weighted inequalities for these operators will be also discussed.

Interpolating bases in spaces of differentiable functions on Cantor-type sets

NECİP ÖZFİDAN

Bilkent University, Ankara, Turkey

ozfidan@fen.bilkent.edu.tr

We construct a basis in spaces of continuous functions and C^1 -functions on Cantor-type sets by using the fundamental polynomials of the first and second kind for the Hermite-Fejér interpolation.

On Hardy classes with variable exponent

VAKHTANG PAATASHVILI

Razmadze Mathematical Institute, Tbilisi, Georgia

kokil@rmi.acnet.ge

Coauthor: V. Kokilashvili

The goal of this talk is to introduce the generalized Hardy classes of analytic and harmonic functions with variable exponent, to present their boundary properties, to give necessary and sufficient conditions for the representation of analytic functions by the Poisson integral with density from variable Lebesgue spaces, to explore the problem of belonging of the Cauchy-type integral with a density from variable Lebesgue spaces to a generalized Hardy class, an extension of the well-known Smirnov theorem, and to give the solution of the Riemann problem in the above-mentioned classes.

Dominated compactness theorem in Banach function spaces and applications

HUMBERTO RAFEIRO

Universidade do Algarve, Faro, Portugal

hrafeiro@ualg.pt

A famous *dominated compactness theorem* due to Krasnosel'skiĭ states that compactness of a regular linear integral operator in L^p follows from that of a majorant operator. This theorem is extended to the case of the spaces $L^{p(\cdot)}(\Omega, \mu, \varrho)$, $\mu\Omega < \infty$, with variable exponent $p(\cdot)$, where we also admit power type weights ϱ . This extension is obtained as a corollary to a more general similar dominated compactness theorem for arbitrary Banach function spaces for which the dual and associate spaces coincide.

Inequality of O'Neil type for convolutions associated with the Laplace-Bessel differential operator and applications

ZAMAN V. SAFAROV

Institute of Mathematics and Mechanics, Baku, Azerbaijan

szaman@rambler.ru

The generalized shift operator T^y is $T^y f(x) = C_{k,\gamma} \int_0^\pi \cdots \int_0^\pi f((x', y')_\alpha, x'' - y'') d\nu(\alpha)$, where $C_{k,\gamma} = \pi^{-\frac{k}{2}} \prod_{i=1}^k \frac{\Gamma((\gamma_i+1)/2)}{\Gamma(\gamma_i/2)}$, $(x', x'') \in \mathbb{R}^k \times \mathbb{R}^{n-k}$, $(x', y')_\alpha = ((x_1, y_1)_{\alpha_1}, \dots, (x_k, y_k)_{\alpha_k})$, $(x_i, y_i)_{\alpha_i} = (x_i^2 - 2x_i y_i \cos \alpha_i + y_i^2)^{1/2}$, and $d\nu(\alpha) = \prod_{i=1}^k \sin^{\gamma_i-1} \alpha_i d\alpha_i$, $1 \leq i \leq k$, $1 \leq k \leq n$. It is well known that T^y is closely related to the Laplace-Bessel differential operator Δ_B . Furthermore, T^y generates the corresponding B -convolution $(f \otimes g)(x) = \int_{\mathbb{R}_{k,+}^n} f(y) (T^y g(x)) (y')^\gamma dy$.

Theorem. 1. Let $1 < p < q < \infty$, $1/p' + 1/q = 1/r$, $f \in L_{p,\gamma}(\mathbb{R}_{k,+}^n)$, $g \in WL_{r,\gamma}(\mathbb{R}_{k,+}^n)$. Then $f \otimes g \in L_{q,\gamma}(\mathbb{R}_{k,+}^n)$ and $\|f \otimes g\|_{L_{q,\gamma}} \leq A_1 \|f\|_{L_{p,\gamma}} \|g\|_{WL_{r,\gamma}}$, where $A_1 = C_{k,\gamma} r' (1+r') (p^{1/q} q^{1/p'} + (p')^{1/q} (q')^{1/p'})$.

2. Let $p = 1$, $1 < q < \infty$, $f \in L_{1,\gamma}(\mathbb{R}_{k,+}^n)$, $g \in WL_{q,\gamma}(\mathbb{R}_{k,+}^n)$. Then $f \otimes g \in WL_{q,\gamma}(\mathbb{R}_{k,+}^n)$ and $\|f \otimes g\|_{WL_{q,\gamma}} \leq A_1 \|f\|_{L_{1,\gamma}} \|g\|_{WL_{q,\gamma}}$, where $A_1 = 2 C_{k,\gamma} r' (1+r')$.

By using an O'Neil-type inequality for rearrangements we obtain a pointwise rearrangement estimate of the B -convolution. As an application, we obtain necessary and sufficient conditions on the parameters for the boundedness of the B -fractional integral operator with rough kernels

$$I_{\Omega,\alpha,\gamma}f(x) = \int_{\mathbb{R}_{k,+}^n} \frac{\Omega(y)}{|y|^{Q-\alpha}} T^y f(x) (y')^\gamma dy$$

from the spaces $L_{p,\gamma}$ to $L_{q,\gamma}$ and from the spaces $L_{1,\gamma}$ to the weak spaces $WL_{q,\gamma}$.

Weighted estimates for the maximal operator in the variable exponent spaces on metric measure spaces

STEFAN G. SAMKO

Universidade do Algarve, Faro, Portugal

ssamko@ualg.pt

We present results on the boundedness of the maximal operator in weighted spaces $L^{p(\cdot)}(X, \rho)$ with variable exponent $p(x)$ on a metric measure space X satisfying the doubling condition. In the case where X is bounded, the weight function belongs to a certain version of a general Muckenhoupt-type condition, which is narrower than the expected Muckenhoupt condition for variable exponent, but coincides with the usual Muckenhoupt class A_p in the case of constant p . For a bounded X we also consider a class of weights of the form $\rho(x) = \prod_{k=1}^n w_k(d(x, x_k))$, $x_k \in X$, where the functions $w_k(r)$ have finite upper and lower indices $m(w)$ and $M(w)$ satisfying the condition $-\frac{m(\mu B)}{p(x_k)} < m(w) \leq M(w) < \frac{m(\mu B)}{p'(x_k)}$, where $m(\mu B)$ is the infimum with respect to $x \in X$ of the lower index of $\mu B(x, \cdot)$. In the case of unbounded X we admit power type weights of the type $w_0[1 + d(x_0, x)] \prod_{k=1}^m w_k[d(x_k, x)]$.

Some of the results are new even in the case of constant p . We also deal with some new notions of upper and lower local dimensions of metric measure spaces.

Several questions of the approximation theory of functions in spaces $L^{p(x)}(E)$

I. I. SHARAPUDINOV

Russian Academy of Sciences, Makhachkala, Russia

sharapud@iwt.ru

Let μ be the Lebesgue measure on a set E and $L^{p(x)}(E)$ the set of measurable functions f on E for which $\int_{E \setminus B(p)} |f(x)|^{p(x)} d\mu(x) < \infty$ and $\text{ess sup}_{x \in B(p)} |f(x)| < \infty$, where $1 \leq p(x)$ is a measurable function essentially bounded on $E \setminus B(p)$ and becoming ∞ on $B(p)$. We consider several problems concerned with approximating functions in $L^{p(x)}(E)$. The topology of space $L^{p(x)}(E)$ have been investigated and it has been proved that if $p(x) \geq 1$, then $L^{p(x)}(E)$ is a normed space on which an equivalent norm is $\|f\|_p(E) = \inf\{\alpha > 0 : \int_E |f(x)|/\alpha^{p(x)} d\mu(x) \leq 1\}$. It is proved if $q(x) = \frac{p(x)}{p(x)-1}$, then the space $L^{q(x)}(E)$ is the dual space to $L^{p(x)}(E)$. It is proved that the Haar system is a basis in $L^{p(x)}([0, 1])$ if the function $p(x)$ satisfies for $x, y \in [0, 1]$ the Dini-Lipschitz condition $|p(x) - p(y)| |\ln|x - y||^{-1} \leq d$.

Let $p = p(x)$ be a measurable, 2π -periodic and essentially bounded function, $L_{2\pi}^{p(x)}$ the space of measurable 2π -periodic functions f for which $\int_{\pi+a}^{\pi+a} |f(x)|^{p(x)} dx < \infty$, where a is an arbitrary real number. We define a norm in $L_{2\pi}^{p(x)}$ by $\|f\|_{p,\pi} = \inf\{\alpha > 0 : \int_{-\pi+a}^{\pi+a} |f(x)|/\alpha^{p(x)} dx \leq 1\}$. The problem about the boundedness in $L_{2\pi}^{p(x)}$ of the operator $f \rightarrow \tilde{f}$ of conjugation given by

$$\tilde{f}(x) = \frac{1}{\pi} \int_{-\pi}^{\pi} \frac{f(t)}{2 \tan \frac{1}{2}(t-x)} dt$$

is considered. It is proved that the trigonometric system is a basis in $L^{p(x)}([-\pi, \pi])$ if the function $p(x) > 1$ satisfies for $x, y \in [-\pi, \pi]$ the Dini-Lipschitz condition $|p(x) - p(y)| |\ln|x - y||^{-1} \leq d$.

On the boundedness of the Hardy-Steklov operator

TAMARA TARARYKOVA

Cardiff University, Wales, UK

tararykovat@cf.ac.uk

Coauthors: V. I. Burenkov & P. Jain

Let a and b be increasing continuous functions defined on $[0, \infty]$ satisfying $a(0) = b(0) = 0$, $a(x) < b(x)$ for $x \in (0, \infty)$ and $a(\infty) = b(\infty) = \infty$. Consider the general Hardy-Steklov operator

$$(Sf)(x) = \frac{1}{b(x) - a(x)} \int_{a(x)}^{b(x)} f(t) dt.$$

H. P. Heinig and G. Sinnamon [*Studia Math.* **129** (1998), 157–177] obtained a Muckenhoupt-type necessary and sufficient conditions ensuring the boundedness of S from one weighted Lebesgue space $L^p(v)$ to another one $L^q(u)$ with arbitrary weight functions u and v .

We give [V. Burenkov, P. Jain & T. Tararykova, *Math. Nachr.* **280** (2007), 1–13]. an alternate (non-Muckenhoupt type) type criterion for the boundedness and obtain a better estimate for the norm of the operator S , which allows to apply this result to investigation of the boundedness, by using a limiting procedure, of the following geometric Steklov operator corresponding to S :

$$(G_S f)(x) = \exp\left(\frac{1}{b(x) - a(x)} \int_{a(x)}^{b(x)} \ln f(t) dt\right), \quad f \geq 0.$$

Multiplicative inequalities for moduli of smoothness of various orders

ASKHAB Y. YAKUBOV

Chechen University, Grozny, Russia

ssamko@ualg.pt

Let X be one of the spaces C , L_p , $p \geq 1$, and $\ell \in \mathbb{R}_+$. We define the modulus of smoothness of order $\ell > 0$ by $\omega_\ell(f, \delta) = \sup_{|h| \leq \delta} \|(E - \tau_h)^\ell f\|_X$.

Theorem. Let $0 < \ell_1 < \ell_2 < \ell_3$. Then $\omega_{\ell_2}(f, \delta) \leq C \cdot \omega_{\ell_1}^{1-\nu}(f, \delta) \cdot \omega_{\ell_3}^\nu(f, \delta)$ for every $\nu \in [0, d]$, $d = \frac{\ell_2 - \ell_1}{\ell_3 - \ell_1}$, $f \in X$.

For the generalized Hölder spaces $H_{\ell_i}^\varphi$, $i = 1, 2, 3...$ with $\ell_1 < \ell_2 < \ell_3$ we obtain the inequality $\|f\|_{H_{\ell_2}^\varphi} \leq C \|f\|_{H_{\ell_1}^\varphi}^{1-\nu} \cdot \|f\|_{H_{\ell_3}^\varphi}^\nu$, where φ is in the generalized Bary-Steckin class $\in \Phi_\lambda^0$. We also obtain a similar inequality for the norms of the spaces $H_\ell^\varphi(\rho) \cap L^p(\rho^0)$ with some weight functions ρ and ρ^0 .

15 Numerical Functional Analysis

Organizers

Pavel E. Sobolevskii (*Universidade Federal do Ceará, Fortaleza, Brazil*)

Allaberen Ashyralyev (*Fatih University, İstanbul, Turkey*)

A note on modified Padé difference schemes

ALLABEREN ASHYRALYEV

Fatih University, İstanbul, Turkey

aashyr@fatih.edu.tr

High order of accuracy modified Padé difference schemes for the approximate solution of the nonlocal boundary value problem for the differential equation

$$v'(t) + Av(t) = f(t) \quad (0 \leq t \leq 1), \quad v(0) = v(\lambda) + \mu, \quad 0 < \lambda \leq 1$$

in an arbitrary Banach space E with the strongly positive operator A are considered. The well-posedness of these difference schemes is established. In applications, the almost coercive stability and the coercive stability estimates for the solutions of difference schemes for the approximate solutions of the nonlocal boundary value problems for parabolic equations are obtained.

On a non-local problem for parabolic-hyperbolic equations with three lines of type changing

ABDUMAUUVLEN S. BERDYSHEV

Uzbek Academy of Sciences, Tashkent, Uzbekistan

berdyshev@mail.ru

Coauthor: N. A. Rakhmatullaeva

In the present paper a problem with non-local condition in a hyperbolic parts of mixed domain for parabolic-hyperbolic equation is considered. The uniqueness of the solution is proved by energy integral method and the existence by the method of integral equations. We consider the equation

$$0 = \begin{cases} u_{xx} - u_y - \lambda y, & (x, y) \in \Omega_0, \\ u_{xx} - u_{yy} + \mu u, & (x, y) \in \Omega_i \ (i = 1, \dots, s) \end{cases} \quad (1)$$

in a domain $\Omega = \Omega_0 \cup \Omega_i \ (i = 1, \dots, s)$, where Ω_0 is domain, bounded by segments AB, BC, CD, DA of straight lines $y = 0, x = 1, y = 1$ and $x = 0$, i.e. the square $\{0 < x < 1, 0 \leq y \leq 1\}$; Ω_2 is a characteristic triangle bounded by the segment AD of the axis y and by the characteristics $AK : x + y = 0, DK : y - x = 1$ of equation (1) outgoing from the points A, D and crossing at $K(-\frac{1}{2}; \frac{1}{2})$; Ω_3 is a characteristic triangle bounded by the segment BC and by characteristics $BM : x - y = 1, CM : x - 2 = -y$ of equation (1) outgoing from the points B, C and crossing at $M(\frac{3}{2}; \frac{1}{2})$; Ω_1 a characteristic triangle bounded by the segment AB and by characteristics $AN : x + y = 0, BN : x - y = 1$ of equation (1) outgoing from the points A, B and crossing at $N(\frac{1}{2}; -\frac{1}{2})$. In equation (1), λ, μ are given real parameters.

Global exponential periodicity for discrete-time Hopfield neural networks with finite distributed delays and impulses

VALÉRY COVACHEV

Sultan Qaboos University, Muscat, Oman

valery@squ.edu.om

Coauthors: H. Akça, Z. Kovacheva & S. Mohamad

Discrete counterpart of a class of Hopfield neural networks with periodic integral impulsive conditions and finite distributed delays is introduced. Mawhin's continuation theorem of coincidence degree theory is used to obtain a sufficient condition for the existence of a periodic solution of the discrete system. By introducing a suitable Lyapunov functional a sufficient condition is obtained for the uniqueness and global exponential stability of the periodic solution.

Numerical solution of a one-dimensional parabolic inverse problem

ABDULLAH SAİD ERDOĞAN

Fatih University, İstanbul, Turkey

aserdogan@fatih.edu.tr

Coauthor: A. Ashyralyev & E. Demirci

We are interested in studying the stable difference schemes for the approximate solutions of the parabolic equation with control parameter

$$\begin{cases} \frac{\partial u(t, x)}{\partial t} = \frac{\partial^2}{\partial x^2} u(t, x) + p(t) u + f(t, x), & x \in (0, 1), \\ t \in (0, T], & u(0, x) = \varphi(x), \quad x \in [0, 1], \\ u(t, 0) = \alpha(t), \quad u(t, 1) = b(t), \quad u(t, x^*) = \gamma(t), & t \in [0, T], \quad x^* \in (0, 1), \end{cases}$$

where $\alpha(t)$, $b(t)$, $\gamma(t)$, $f(t, x)$ and $\varphi(x)$ are given sufficiently smooth functions and $u(t, x)$, $p(t)$ are unknown functions.

In the present paper the first and second orders of accuracy difference schemes for approximately solving this problem are presented. A procedure of modified Gauss elimination method is used for solving these difference schemes. The method is illustrated by numerical examples.

Numerical solution of nonlocal boundary value problems for elliptic-parabolic equations

OKAN GERÇEK

Fatih University, İstanbul, Turkey

ogercek@fatih.edu.tr

Coauthor: A. Ashyralyev

A numerical method is proposed for solving the elliptic-parabolic partial differential equations with nonlocal boundary conditions. The first and second order of accuracy difference schemes are presented. The method is illustrated by numerical examples.

Positivity of difference operators

ESAT GEZGİN

Fatih University, İstanbul, Turkey

esatgezgin198@hotmail.com

Coauthor: A. Ashyralyev

In the present paper the positivity of a difference operator approximating the differential operator A defined by

$$Au = a(x) \frac{d^4 u}{dx^4} + \delta u$$

with the boundary conditions in $C[0, 1]$ is established. Here $a(x)$ is a smooth function defined on the segment $[0, 1]$, $a(x) \geq a > 0$, and $\delta > 0$.

On a difference scheme of second order of accuracy for hyperbolic equations

MEHMET EMİR KÖKSAL

Fatih University, İstanbul, Turkey

mekoksal@fatih.edu.tr

Coauthor: A. Ashyralyev

We are interested in studying high order of accuracy two-step absolute stable difference schemes for the approximate solutions of the initial value problem

$$\frac{d^2 u(t)}{dt^2} + A(t)u(t) = f(t), \quad 0 \leq t \leq T, \quad u(0) = \varphi, \quad u'(0) = \psi$$

in a Hilbert space H with self-adjoint positive definite operators $A(t)$. In the present paper a new difference scheme of second order of accuracy for approximately solving this initial-value problem is presented. Stability estimates for the solution of this difference scheme are established.

Averages of orthogonal polynomials on two intervals

ALEXEY L. LUKASHOV

Saratov State University, Russia

lukashov@fatih.edu.tr

Uniform asymptotics of orthogonal polynomials on several arcs in the complex plane were given by H. Widom (1969) and have the following form: $P_n(z)C(E)^n\Phi^{-n}(z) \sim F_n(z)$, where P_n is an orthogonal polynomial, $C(E)$ is the logarithmic capacity of the support of the orthogonality measure, $\Phi(z)$ is the exponential of the complex Green function of the domain outside of E .

The main result is the uniform asymptotics of special averages of the left hand side expression from above that does not depend on n for the case $E = [-1, \alpha] \cup [\beta, 1]$.

WKB asymptotics for a linear Schroedinger type O.D.E. with a turning point and a singular point

MINORU NAKANO

Keio University, Yokohama, Japan

nakano_math_keio@yahoo.co.jp

The existence of domains of asymptotic solutions of O.D.E. is determined by Stokes curves, which are very complicated with respect to numbers and ranks of turning points and/or singular points. We consider a typical Schroedinger type O.D.E. with a turning point and a singular point by applying the complex WKB analysis.

General representation of solutions to the system of differential equations of hemitropic elasticity and its applications

DAVID NATROSHVILI

Tbilisi State University, Akhaltsikhe, Georgia

natrosh@hotmail.com

Coauthors: L. Giorgashvili & S. Zazashvili

Technological and industrial developments, and also great success in biological and medical sciences require to use more generalized and refined models for elastic bodies. In a generalized solid continuum, the usual displacement field has to be supplemented by a microrotation field. Such materials are called micropolar or Cosserat solids. They model composites with a complex inner structure whose material particles have 6 degrees of freedom (3 displacement components and 3 microrotation components). The corresponding second order partial differential equations generate a 6×6 matrix differential operator which has a very complex structure.

Experiments have shown that micropolar materials possess quite different properties in comparison with the classical elastic materials. For example, in noncentrosymmetric micropolar materials (which are called also hemitropic or chiral materials) there propagate the left-handed and right-handed elastic waves. Materials may exhibit chirality on the atomic scale, as in quartz and in biological molecules—DNA, as well as on a large scale, as in composites with helical or screw—shaped inclusions, certain types of nanotubes, bone, fabricated structures such as foams, chiral sculptured thin films and twisted fibers.

Here we derive general representation formulas for solutions to the system of differential equations of hemitropic elasticity by means of harmonic and metaharmonic functions. With the help of these formulas we solve boundary value and transmission problems for homogeneous and piecewise homogeneous hemitropic elastic bodies when boundaries and interface surfaces are spheres. The solutions are constructed explicitly in the form of uniformly and absolutely convergent Fourier-Laplace series. Some particular practical problems (e.g., J. D. Eshelby type contact problems) are analyzed in detail.

Application of pseudodifferential equations in the theory of piezoelectric-metallic composites

DAVID NATROSHVILI

Georgian Technical University, Tbilisi, Georgia

natrosh@hotmail.com

Coauthors: T. Buchukuri & O. Chkadua

We study the following mathematical problem related to engineering applications. Given is a three-dimensional composite consisting of a piezoelectric (ceramic) matrix with metallic inclusions (electrodes) and containing interior or interface cracks. We derive a linear model for the interaction of the corresponding 4-dimensional thermoelastic field in the metallic part and 5-dimensional thermoelectroelastic field in the piezo-ceramic part.

The main difficulty in the modelling is to find appropriate boundary and transmission conditions for the composed body with cracks. The mathematical analysis includes then the study of existence, uniqueness and regularity of the resulting elliptic boundary-transmission problem assuming the metallic and ceramic materials occupy smooth or polyhedral domains.

With the help of the indirect boundary integral equations method we reduce the complex transmission problem to the equivalent strongly elliptic system of pseudodifferential equations involving pseudodifferential operators on manifolds with boundary. The solvability and regularity of solutions to these boundary integral equations and the original transmission problem are analyzed in Sobolev-Slobodetski (Bessel potential) H_p^s and Besov $B_{p,t}^s$ spaces. This enables us to investigate also stress singularities which appear near crack edges and also near zones where the boundary conditions change and where the interface meets the exterior boundary.

We show that the order of the singularity is related to the eigenvalues of the symbol matrices of the corresponding pseudodifferential operators and study their dependence on the material constants of the composite. Oscillating stress singularities are analyzed as well.

A note on difference schemes of second order of accuracy for hyperbolic-parabolic equations

YILDIRIM ÖZDEMİR

Fatih University, İstanbul, Turkey

yozydemir@fatih.edu.tr

Coauthor: A. Ashyralyev

A second order of accuracy difference scheme for approximately solving the multipoint non-local boundary value problem for the differential equation

$$\begin{cases} \frac{d^2 u(t)}{dt^2} + Au(t) = f(t) & (0 \leq t \leq 1), & \frac{du(t)}{dt} + Au(t) = g(t) & (-1 \leq t \leq 0), \\ u(-1) = \sum_{j=1}^N \alpha_j u(\mu_j) + \sum_{j=1}^L \beta_j u'(\lambda_j) + \varphi, & \sum_{j=1}^N |\alpha_j| \leq 1, & \sum_{j=1}^L |\beta_j| \leq 1, & 0 < \mu_j, \lambda_j \leq 1 \end{cases}$$

in a Hilbert space H with the self-adjoint positive definite operator A is presented. Stability estimates for the solution of this difference scheme are established. In applications, stability estimates for the solutions of the difference schemes of the mixed type boundary value problems for hyperbolic-parabolic equations are obtained.

Numerical solutions of Bitsadze-Samarskii problem for elliptic equations

ELİF ÖZTÜRK

Fatih University, İstanbul, Turkey

eozyturk@fatih.edu.tr

Coauthor: A. Ashyralyev

Bitsadze-Samarskii type problem for the elliptic equation

$$-\frac{d^2 u(t)}{dt^2} + Au(t) = f(t) \quad (0 \leq t \leq 1), \quad u(0) = \varphi, \quad u(1) = \sum_{m=1}^n \alpha_m u(\lambda_m) + \psi$$

in a Hilbert space H with the self-adjoint positive definite operator A is considered. A numerical method is proposed for solving this problem with nonlocal boundary conditions. Second and fourth order of accuracy difference schemes are presented. A modified Gauss elimination method is used for solving these difference schemes in the case of a two-dimensional elliptic partial differential equations. The method is illustrated by numerical examples.

Weak maximal regularity for abstract hyperbolic problems in function spaces

JAVIER PASTOR

University of Valencia, Spain

Vicente.Pastor@uv.es

Coauthors: A. Ashyralyev, M. Martínez & S. Piskarev

This paper is devoted to the numerical analysis of abstract hyperbolic differential equations in $C([0, T]; E)$, $C^\alpha([0, T]; E)$ spaces. The maximal regularity inequalities play an important role in the theory of differential equations (Ashyralyev and Sobolevskii (1994), Piskarev (2005)). First of all, we show that for the second order equations the maximal regularity inequality does not exist in the spaces $C([0, T]; E)$, $C([0, T]; E^\alpha)$, $C^\alpha([0, T]; E)$, and $L^p([0, T]; E)$. Therefore, for the solutions of difference scheme for the second order differential equations, only the weak maximal regularity estimates are established. The presentation uses general approximation scheme in Banach spaces and is based on C_0 -cosine operator theory and functional analysis.

On well-posedness of parabolic differential and difference equations

YAŞAR SÖZEN

Fatih University, İstanbul, Turkey

ysozen@fatih.edu.tr

Coauthor: A. Ashyralyev

We consider the parabolic differential equation

$$u'(t) + \mathbf{A}u(t) = f(t), \quad -\infty < t < \infty.$$

It is well known that the abstract parabolic differential equation in a general Banach space \mathbf{E} with the strongly positive operator \mathbf{A} is ill-posed in the Banach space $C(\mathbb{R}, \mathbf{E})$. We will establish the well-posedness of this equation in the Hölder space $C^\beta(\mathbb{R}, \mathbf{E}_\alpha)$. The well-posedness of the Rothe difference scheme in $C^\beta(\mathbb{R}_\tau, \mathbf{E}_\alpha)$ will be proved. As application, new coercivity inequalities will be obtained for the solutions of the parabolic difference equations.

16 Integrable Systems

Organizers

Metin Gürses (*Bilkent University, Ankara, Turkey*)

Ismagil T. Habibullin (*Bilkent University, Ankara, Turkey*)

Initial value problem and traveling wave solution for spherical Liouville equation

BARIŞ ATEŞ

İzmir Institute of Technology, Turkey

barisates@iyte.edu.tr

Coauthor: O. K. Pashaev

Following the method of spherical means for solution of multi-dimensional wave equation, the initial value problem in 3 space dimensions is reducible to the one dimensional wave equation on half line. By using Backlund transformation in (r, t) , we relate this equation with the spherical Liouville equation $\Phi_{tt} - c^2(\Phi_{rr} + \frac{2}{r}\Phi_r) = -\frac{4c^2}{r}e^{r\Phi}$. Integrating the Backlund transformation, the general solution of the Liouville equation with spherical symmetry is obtained. It allows us to solve the initial value problem for the Liouville equation in the special case $\Phi(r, 0) = \phi(r)$, $\Phi_t(r, 0) = 0$ in the form

$$\Phi(r, t) = \frac{(r + ct)\phi(r + ct) + (r - ct)\phi(r - ct)}{2r} - \frac{1}{r} \ln \cosh^2 \int_{r-ct}^{r+ct} e^{\rho\phi(\rho)/2} \frac{d\rho}{\sqrt{2}}.$$

By direct integration of the traveling wave form of the equation, nonlinear spherical waves are constructed.

Spinor representations in integrability of some field-theoretic equations

VALENTINA A. GOLUBEVA

Russian Academy of Sciences, Moscow, Russia

goloubeva@yahoo.com

In contemporary physics and geometry, the important problem concerning the integrability of different field-theoretic differential equations arises. Among them are the Dirac, twistor and Killing equations on Weyl manifolds with spin structure, Calogero-Moser-Sutherland equations, generalized Knizhnik-Zamolodchikov equations associated with non one-laced root systems. The CS and GKZ models related to the mentioned root systems are naturally multiparametric: one parameter corresponds, for example, to the scattering of particles, the other one to their reflections. Other parameters are also possible. Since the solutions to corresponding equations change their phases, they have to be described in terms of spinorial representations of corresponding symmetry algebra of the model.

In this talk the spinorial structures connected to the equations mentioned on Weyl spin manifolds and their integrability conditions based on Schrödinger-Lichnerovich formula will be considered. Then the integrability conditions will be given for the case of two-parametric CM and multiparametric GKZ equations. Some spinorial solutions of these integrability systems will be considered and given explicitly.

Classification of integrable discrete equations

ISMAGIL HABIBULLIN

Bilkent University, Ankara, Turkey

habib@fen.bilkent.edu.tr

Differential-difference equation of the form

$$\frac{d}{dx}u(n+1) = f\left(u(n), u(n+1), \frac{d}{dx}u(n)\right) \quad (1)$$

is of hyperbolic type if f depends essentially upon its third argument, i.e. if $\partial f(x, y, z)/\partial z$ does not vanish identically. Equation (1) is called Darboux integrable if two functions $F(n, x, t(n), t(n \pm 1), t(n \pm 2), \dots)$ and $I(n, x, t(n), t_x(n), t_{xx}, \dots)$ of a finite number of arguments exist such that $D_x F = 0$ and $DI = I$, where D_x is the operator of total differentiation with respect to x , and D is the shift operator: $Dp(n) = p(n+1)$. In this talk an algebraic criterion of integrability and the problem of classification of integrable equations of the form (1) will be discussed.

Invariance, integrability and linearization of a generalized modified Emden type equation

SATYENDRA N. PANDEY

Nehru National Institute of Technology, Allahabad, India

snp@mnnit.ac.in

Coauthor: M. Lakshmanan

In this paper, we consider a generalized modified Emden type equation (GMEE) of the form $\ddot{x} + (k_1x + k_2)\dot{x} + k_1^2x^3/9 + k_1k_2x^2/3 + \lambda_1x + \lambda_2 = 0$, where k_i and λ_i , $i = 1, 2$, are arbitrary parameters, which includes several physically important oscillators such as the simple harmonic oscillator, displaced damped harmonic oscillator, Emden type equation and its hierarchy, and so on, and investigate the Lie point symmetries, integrability and linearization. We show that the equation admits eight Lie point symmetry generators. By using extended Prolle-Singer procedure, we derive integrating factors and integrals of motion of this equation and obtain the general solution. We also explore its general solution by transforming it to a linear third order ordinary differential equation. Finally, we show that under certain parametric choice the GMEE exhibits unusual nonlinear dynamical properties.

Solutions of the extended Kadomtsev-Petviashvili-Boussinesq equation by the Hirota direct method

ASLI PEKCAN

Bilkent University, Ankara, Turkey

asli@fen.bilkent.edu.tr

We show that we can apply the Hirota direct method to some non-integrable equations. For this purpose, we consider the extended Kadomtsev-Petviashvili-Boussinesq (eKPBo) equation with M variables, which is

$$(u_{xxx} - 6uu_x)_x + a_{11}u_{xx} + 2 \sum_{k=2}^M a_{1k}u_{xx_k} + \sum_{i,j=2}^M a_{ij}u_{x_i x_j} = 0,$$

where $a_{ij} = a_{ji}$ are constants and $x_i = (x, t, y, z, \dots, x_M)$. We give the results for $M = 3$ and a detailed work for $M = 4$. Then we will generalize the results to any integer $M > 4$.

Chern and de Rham-Hodge aspects of the Gromov type differential relations and applications to multi-dimensional integrable systems

ANATOLIY K. PRYKARPATSKY

National Academy of Sciences, Lviv, Ukraine

`pryk.anat@ua.fm`

The differential-geometric aspects of generalized de Rham-Hodge complexes naturally related with integrable multi-dimensional differential systems of Gromov type, as well as the geometric structure of Chern characteristic classes are studied. Special differential invariants of the Chern type are constructed, their importance for the integrability of multi-dimensional nonlinear differential systems on Riemannian manifolds is discussed. An example of the three-dimensional Davey-Stewartson type nonlinear integrable differential system is considered, its Cartan type connection mapping and related Chern type differential invariants are analyzed. Some applications to integrable multi-dimensional dynamical systems are considered.

Integrable discrete systems on \mathbb{R}

BURCU SİLİNDİR

Bilkent University, Ankara, Turkey

`silindir@fen.bilkent.edu.tr`

Coauthors: M. Blazsak, M. Gürses & B. M. Szablikowski

In order to obtain integrable discrete systems, we first define a grain structure on \mathbb{R} . By the use of this grain structure we construct an algebra of differential operators. Introducing an inner product, trace functional, Lie derivative and an R -matrix structure on this algebra, the Gelfand-Dikii Formalism and hence the Lax formulation of several integrable systems of evolution equations can be given. We find two Hamiltonian operators for each system and we show that these systems are bi-Hamiltonian. Furthermore the continuous limits of these bi-Hamiltonian systems are also discussed.

Integrable nonlinear equations on a circle

KOSTYANTYN ZHELTUKHIN

Middle East Technical University, Ankara, Turkey

`zheltukh@metu.edu.tr`

The concept of integrable boundary value problems for soliton equations on \mathbb{R} and \mathbb{R}_+ is extended to bounded regions enclosed by smooth curves. Classes of integrable boundary conditions on a circle for the Toda lattice and its reductions are found.

17 General Session

Organizer

A. Okay Çelebi (*Middle East Technical University, Ankara, Turkey*)

Cohen p -nuclear multilinear mappings

DAHMANE ACHOUR

M'sila University, Algeria

dachourdz@yahoo.fr

In this communication, we introduce and study a new concept of multilinear summability operators, which we call “Cohen p -nuclear multilinear operators”. We give the Pietsch domination theorem for this notion. Also we compare the notion of p -nuclear multilinear operators with the class of Cohen strongly p -summing multilinear operators, r -dominated and q -integral. We give some theorems of composition for this class of operators.

Series solution for non-linear differential operators using modified homotopy analysis method

FATHI M. ALLAN

United Arab Emirates University, Al Ain, UAE

f.allan@uaeu.ac.ae

Construction of series solutions for nonlinear differential operators will be considered. A new algorithm will be used to construct the homotopy polynomials. Based on this algorithm, the issue of convergence and the error analysis of the method as applied to a special class of ordinary differential equations will be addressed.

Self similar approach for visualization of nonlinear processes

MERSAID ARIPOV

National University of Uzbekistan, Tashkent, Uzbekistan

mirsaidaripov@mail.ru

In this paper we demonstrate the possibilities of the self similar, approximately self similar approaches to the studying of properties of nonlinear reaction-diffusion systems under action of a convective transfer. An influence of parameters of the reaction-diffusion systems to an evolution of the process is studied. It is proved that there exist some values of parameters when the effect of finite velocity of perturbations, localization of solution, onside localization, the effect of “wall”, blow up, blow up localization. For construction of self similar system the method of the nonlinear splitting is offered [M. Aripov, *ZAMM* **80** (2000), 767–768]. Numerical analysis and visualization of nonlinear reaction-diffusion processes for different value of parameters using self similar approach were carried out. The results of numerical experiments have showed the effectivity of self similar approach to studying of the nonlinear reaction diffusion processes.

Structural stability for nonlinear shallow water waves

ÜLKÜ DİNLEMEZ ATEŞ

Gazi University, Ankara, Turkey

ulku@gazi.edu.tr

Coauthor: A. O. Çelebi

In this talk we will discuss the structural stability of an initial value problem defined for the equation

$$u_t - u_{txx} + \alpha uu_x = \beta u_x u_{xx} + uu_{xxx}, \quad (1)$$

where α, β are constants, $x \in \mathbb{R}$, and $t \in \mathbb{R}^+$. For choices of α and β , (1) is used for nonlinear shallow water waves. Particularly for the choices $\alpha = 4, \beta = 3$ and $\alpha = 3, \beta = 2$, Degasperis-Procesi and Camassa-Holm equations are attained respectively.

Noether's theorem on time scales

ZBIGNIEW BARTOSIEWICZ

Białystok Technical University, Poland

bartos@pb.bialystok.pl

Coauthors: G. S. F. Frederico & D. F. M. Torres

The classical theorem of Emmy Noether (1918) on calculus of variations says that for any variational symmetry of a minimization problem there exists a conserved quantity along the respective Euler-Lagrange extremals. We extend this theorem to the calculus of variations on arbitrary time scales. The differential calculus on time scales unifies differential and difference calculus via the concept of delta derivative, which behaves like the standard derivative $f'(t)$ for the continuous time and like the difference $f(t+1) - f(t)$ for the discrete time. The concept of integral on time scales extends the standard integral and the finite sum. We construct the quantity that is preserved by the variational symmetry. It contains an extra term not present in the classical statement in the continuous time. This term depends on the graininess of the time scale and vanishes when the graininess is 0 (continuous time). The proof of the result is done in two steps. First we prove the theorem for the case when the symmetry transformations do not change time (without transforming the independent variable). Then, using time reparameterization, we obtain the Noether's theorem on time scales in its general form.

Variational study of nonlinear problem of contact without friction between two elastic bodies

BENYATTOU BENABDERAHMANNE

University of Laghouat, Algeria

b.benabderahmanne@mail.lagh-univ.dz

Coauthors: B. Nouri & H. A. Tedjani

In this work, we consider a static problem of contact without friction between two deformable bodies having a linear or nonlinear behaviour law. By using Green's formula and Korn's inequality, we establish two variational formulations of the problem considered. The first formulation, noted P_1 , depends only on the field of displacement, however the second, noted P_2 , depends only on the tensor of constraint. By using a theorem of Stampacchia we prove that the two variational problems P_1 and P_2 have only one solution. Later, we study the relation between the solutions of variational problems P_1, P_2 and the problem P . If the law of behaviour is linear, a new mixed variational formulation is given for the problem P , where the unknowns are the field of displacement u and the function λ , where λ represents the normal component of the constraint tensor on Γ_3 , knowing that the tangential component of the tensor of constraint is null on Γ_3 . By using theorems of Lax-Milgram and Stampacchia, we prove that the linear problem has a unique solution.

On determination of Green's tensor for a granular elastic medium with application to wave propagation in a random medium

RABIN K. BHATTACHARYYA

Calcutta University, India

rabindrakb@yahoo.com

Coauthor: G. Chattopadhyay

The equation $LV = f$, where L is a linear differential operator involving randomly variable field parameters, V the field vector and f the source term, is considered. An integro-differential equation governing the mean field quantity $\langle V \rangle$ is derivable by the use of smooth perturbation technique. The kernel of the deterministic operator equation is the Green's tensor appropriate to the field equations representing the granular elastic medium. This is evaluated in the form of Fourier integrals; the exact evaluation is carried out to obtain the 36 components of the Green's tensor. The problem of wave propagation in the random granular elastic medium is then carried out with the help of Green's tensor. Theoretical and also numerical computational analyses are carried out to explain the effect of random inhomogeneities of the medium on the propagation of waves. It has been shown that the body waves attenuate as they propagate in the medium

Approximate methods for evaluating a new class of hypersingular integral equations

ILYA V. BOYKOV

Penza State University, Russia

boikov@diamond.stup.ac.ru

Coauthor: E. S. Ventsel

The presentation is devoted to a numerical evaluation of a new class of hypersingular integral equations which include hypersingular, singular and weak-singular integrals. We examine the equations

$$a(t)x(t) + \int_{-1}^1 \frac{b(t, \tau)x(\tau)}{(\tau - t)^p} d\tau + \int_{-1}^1 \frac{c(t, \tau)x(\tau)}{\tau - t} d\tau + \int_{-1}^1 d(t, \tau) \ln |\tau - t| d\tau + \int_{-1}^1 h(t, \tau)x(\tau) d\tau = f(t)$$

and

$$\begin{aligned} a(t)x(t) + \iint_D \frac{b(t, \tau)x(\tau)}{r(t, \tau)^{2p}} d\tau_1 d\tau_2 + \iint_D \frac{\varphi(t, \Theta)(t, \tau)x(\tau) d\tau_1 d\tau_2}{r(t, \tau)^2} \\ + \iint_D d(t, \tau) \ln(r(t, \tau))x(\tau) d\tau_1 d\tau_2 + \iint_D h(t, \tau)x(\tau) d\tau_1 d\tau_2 = f(t). \end{aligned}$$

Here $D = [-1, 1]^2$, $t = (t_1, t_2)$, $\tau = (\tau_1, \tau_2)$, $\Theta = (t - \tau)/r(t, \tau)$, $r(t, \tau) = ((\tau_1 - t_1)^2 + (\tau_2 - t_2)^2)^{1/2}$. These equations are often used in simulation in physics and technical problems, such as antenna theory, multi-dimensional Prandte equation, theory of wave-conductors, etc. We propose a spline-collocation method for solution of these equations. We also describe parallel methods for evaluating these equations. Theoretical estimates of the convergence rate and of the errors for these methods are obtained. These estimates coincide with the results of numerical experiments.

A study of buoyancy driven unsteady viscous flow in the presence of applied magnetic field

PALLATH CHANDRAN

Sultan Qaboos University, Al Khod, Oman

chandran@squ.edu.om

Coauthor: N. C. Sacheti

Buoyancy driven flows of electrically conducting fluids near vertical surfaces under the influence of externally applied magnetic field have received extensive attention in the literature due to vast number of applications. Such flows are encountered in several industrial and technological fields such as aerospace, biomedical, chemical and nuclear engineering. In this paper, we report our analysis of a particular class of unsteady developing hydromagnetic flow near a uniformly moving or an accelerating impermeable surface. Analytical solutions of the governing fluid dynamical equations of fluid motion have been obtained subject to constant heat flux condition at the bounding surface. The influence of several key fluid dynamical parameters on the velocity and temperature profiles in the boundary layer has been investigated in detail.

Nonlinear and oblique boundary value problems for Lamé's equations

MOURAD DILMI

Université Jean Monnet, Saint-Etienne, France

mouraddil@yahoo.fr

Coauthors: H. Benseridi & B. Merouani

In this paper, we prove the existence and the regularity of the solution in 2D and 3D of the boundary value problem $-Lu + \alpha u = f$ with Dirichlet and nonlinear boundary conditions $-\sigma(u)\nu + P(u) \in \beta(u)$, where β is a monotone maximal graph such that $0 \in \beta(0)$, and α is a positive real number we will further make more precise. To achieve this goal, we consider the approximate problem in which β is replaced by Yosida's approximation β_ξ , and we solve this last problem by using Brézi's contraction method.

On Hilbert's 16th and Smale's 13th problems

VALERY A. GAIKO

Belarusian State University of Informatics and Radioelectronics, Minsk, Belarus

vlrgk@yahoo.com

We establish the global qualitative analysis of planar polynomial dynamical systems and suggest a new geometric approach to solving Hilbert's sixteenth problem on the maximum number and relative position of their limit cycles in two special cases of such systems. First, using geometric properties of four field rotation parameters of a new canonical system, we present a proof of our earlier conjecture that the maximum number of limit cycles in a quadratic system is equal to four and the only possible their distribution is $3 : 1$. Then, by means of the same geometric approach, we solve the problem for Liénard's polynomial system (in this special case, it is considered as Smale's thirteenth problem). Besides, generalizing the obtained results, we present a solution of Hilbert's sixteenth problem on the maximum number of limit cycles surrounding a singular point for an arbitrary polynomial system and, applying the Wintner-Perko termination principle for multiple limit cycles, we develop an alternative approach to solving the problem. By means of this approach, we give another proof of the main theorem for a quadratic system and complete the global qualitative analysis of a generalized Liénard's cubic system with three finite singularities. We discuss also some different approaches to the problem.

Maximum entropy principle for hierarchical dynamical systems

IGOR M. GAISSINSKI

Technion, Haifa, Israel

igorg@aerodyne.technion.ac.il

Coauthor: V. Y. Rovenski

The maximum entropy (or minimum information) principle is used to derive the equations of the motion for the phase distribution as constraints on the variation problem. The classical Liouville equation is derived when the constraint is the first order linear differential equation. Maximum entropy replaces the fundamental postulates represented by Newton's laws. The new approach opens the possibility to derive more advanced equations of motion by removing constraints. The maximum entropy principle is developed for a turbulence problem determined by hierarchical dynamical systems. The analytical solution is found to Fokker-Planck equation based on the maximum entropy principle for N -dimensional hierarchical dynamical system.

Unique solvability of the Cauchy problem for Hopf equation in 2 dimensions

V. I. GISHLARKAEV

Chechen University, Grozny, Russia

zloy_chechen@list.ru

We are interested in the abstract analog $du(t)/dt + Au(t) + B(u(t)) = f(t)$, $t \in [0, T]$, $T < \infty$, $U|_{t=0} = u_0$ of the system of Navier-Stokes equations, where $A : H \rightarrow H$, $\tilde{B}(\cdot, \cdot) : H \times H \rightarrow H^{-S}$, $B(u) = \tilde{B}(u, u)$, and H is a Hilbert space. The initial data are random elements in H with probability distribution $\mu(du_0)$. The following Hopf equation describes the evolution of the characteristic function of the probability distribution of the velocity of the moving fluid:

$$\frac{\partial \chi}{\partial t}(t, v) + i \int \langle A(w) + B(w) - f(t), v \rangle e^{i\langle w, v \rangle} \mu(t, dw) = 0 \quad \mu(0, dw) = \mu(dw). \quad (1)$$

There exists a solution to problem (1) and we have uniqueness in the class of measures such that $\int_0^T \int \|u\|^2 e^{c\|u\|^4} \mu(t, du) dt < \infty$ ([M. J. Vishik & A. V. Fursiov, *Mathematical Problems of Statistical Hydromechanics*, Kluwer, 1988]).

Theorem. There exists a unique solution to problem (1) in the class of measures that are absolutely continuous with respect to μ .

Using periodic wavelets to solve non-periodic boundary value problems

MOHAMED ALI HAJJI

United Arab Emirates University, Al-Ain, UAE

mahajji@uaeu.ac.ae

In this paper, we describe how wavelets may be used to solve partial differential equations. In particular, we describe how periodic wavelets on the interval can be used to solve non-periodic boundary value problems such as Dirichlet boundary conditions. Non-periodic boundary conditions are incorporated using the capacitance matrix method. Comparison is made against other well known numerical schemes such as the finite-difference method. The results show the wavelet method outperforms the finite difference method.

The absence of boundary problems for differential equations on a segment with nonempty spectrum

BALTABEK E. KANGUZHIN

Centre of Physical and Mathematical Researches, Almaty, Kazakhstan

kanbalta@mail.ru

Coauthors: B. T. Kalmenov & T. S. Kalmenov

We prove the existence of at least one eigenvalue involving the infinity. Thus we show that we have no ordinary differential operator with nonempty finite spectrum. We consider

$$L_Q y(x) = y^{(n)}(x) + \sum_{k=0}^{n-1} p_k(x) y^{(k)}(x)$$

in the Hilbert space $L_2(0, 1)$ with sufficiently smooth coefficients $p_k(x)$ on $[0, 1]$ and general boundary conditions $\sum_{j=0}^{n-1} (a_{jk} y^{(j)}(0) + b_{jk} y^{(j)}(1)) = 0$, $k = 1, \dots, n$, where a_{jk}, b_{jk} are arbitrary numbers. It is known that if the operator L_Q in the space $L_2[0, b]$ has a bounded inverse, then it is a completely continuous operator.

Theorem. Suppose there exists a number c_1 such that $|p_j^{(k)}(x)| < c_1$ for all $j = 0, 1, \dots, n-1$, for all k , for all $x \in [0, 1]$. If the operator L_Q has at least one eigenvalue, then the spectrum of the operator L_Q is an infinite set.

On isomorphisms of cartesian products of l -Köthe spaces

ERDAL KARAPINAR

İzmir University of Economics, Turkey

erdal.karapinar@ieu.edu.tr

Prada in 1984 proved that a complemented subspace of the product $E \times F$ of Frechet spaces E and F , with the property that $L(E, F) = K(E, F)$, is isomorphic to the product $E_0 \times F_0$ of complemented subspaces E_0 of E and F_0 of F . In this short manuscript, we consider this result on the class of cartesian product of Köthe spaces $E_0^{l_2}(a) \times E_\infty^{l_2}(b)$.

Symplectic and multi-symplectic Lobatto methods for the “good” Boussinesq equation

BÜLENT KARASÖZEN

Middle East Technical University, Ankara, Turkey

bulent@metu.edu.tr

Coauthor: A. Aydın

We construct a symplectic and multi-symplectic integrators for the “good” Boussinesq equation using a two-stage Lobatto IIIA-IIIB pair as a partitioned Runge-Kutta method. Some numerical results have been given for the evolution of the solitary waves. Numerical results confirm that the solitary waves of the “good” Boussinesq equation are sensitive to the initial amplitude. Different phenomena have been observed for various ranges of parameters. Both integrators simulate well the solitary waves in long time by preserving conserved quantities like energy and momentum.

Existence of weak solutions of the g -Kelvin-Voigt equations

MERYEM KAYA

Gazi University, Ankara, Turkey

meryemk@gazi.edu.tr

Coauthor: A. O. Çelebi

In this talk we present the existence of weak solutions of the g -Kelvin-Voigt equations with the Dirichlet boundary condition. The 2D g -Kelvin-Voigt equations have the form

$$u_t + \frac{\alpha}{g}(\nabla g \cdot \nabla)u_t - \frac{\alpha}{g}(\nabla \cdot g \nabla u_t) + \frac{\nu}{g}(\nabla g \cdot \nabla)u - \frac{\nu}{g}(\nabla \cdot g \nabla u) + \nabla p + u \cdot \nabla u = f(x)$$

in $\Omega \times (0, T]$ with the continuity equation $\nabla \cdot (gu) = 0$ in $\Omega \times (0, T]$, where g is a suitable smooth real valued function defined on a bounded domain $\Omega \subset \mathbb{R}^2$, $\alpha > 0$, $\nu > 0$. In the proof we use the well-known Faedo-Galerkin method.

An application of a stochastic model for predicting earthquake occurrences in Iran

SHAGHAYEGH KORDNOURIE

Islamic Azad University, Tehran, Iran

shaghayeghkordnourie@yahoo.com

Coauthors: H. Mostafaei & E. Pasha

There are many statistical models for investigating earthquake occurrences. One of these models is a semi-Markov model that by its application, the probability of great earthquakes in three dimensions of space, time and magnitude can be obtained. This model assumes that the successive earthquakes in the same structural discontinuity are dependent events that are influenced by the elapsed time interval between them. The Zagros fold-thrust belt, which is one of the youngest and movable areas of the continental contact, is chosen for this study. For predicting the probability of earthquake occurrence in this region by the semi-Markov model, 440 earthquake data from 1900 to 2007 with magnitude $M \geq 5$ Richter that occurred were used, and the transition probability and holding time distribution and interval transition probabilities for region to region and magnitude to magnitude were obtained. By this way, we can examine the earthquake occurrences in the investigated area in a quantitative way.

Using spectral analysis to estimate the quality factor Q : Application on a vertical seismic profile

DJEDDI MABROUK

Université de Boumerdès, Algeria

djeddi-m1912@umbb.dz

Coauthors: M. A. Aitouche & H. Kerchouche

The processing of vertical seismic profile (VSP) data involves two operative levels: an academic primary step for separating the upgoing wave field from the downgoing wave field, knowing that only the first one presents such interest, because it is related to the reflections coming from the seismic horizons. When the upgoing wave field is acquired, it allows one to calculate discriminating parameters through oriented and subtle operations during the seismic processing. These parameters characterize the formations and can constitute some basic hypotheses for other operations in the future. If the law of attenuation is expressed mathematically under an exponential form, the coefficient α involves several other parameters like velocity, frequency and quality factor Q . Expressed in decibels (dB), the magnitude spectrum of each trace takes a linear form with respect to the frequency variable. The slope is closely related to the coefficient of attenuation α which can be estimated in a specified frequency band. Therefore it is

possible to estimate the corresponding quality factor. The method has been applied on data coming from the re-processing of a VSP for an IR1bis well. For every level of the depth scale, the slope is expressed in dB/Hz for simple time in ms. Some values of the quality factor have been calculated. They are located around the value $Q = 100$, which denotes the existence of a strong attenuation. Notice that quality factor does not vary widely with depth.

Extreme points in the set of topological left invariant means

H. P. MASIHA

Toosi University of Technology, Tehran, Iran

masiha@kntu.ac.ir

In this lecture, the extreme points in the set of all topological left invariant means on a topological left amenable locally compact semigroup will be characterized. It is known that, in the discrete semigroup situation, each multiplicative topological left invariant mean (if any) is an extreme point in the set of all topological left invariant means. We obtain a partial analogue of its converse for discrete semigroups which is due to Granirer. We also characterize the extreme points as approximately multiplicative topological left invariant means.

On the basicity in $L_p(0, 1)$ ($1 < p < \infty$) of the eigenfunctions of a periodic (or anti-periodic) problem

HAMZA MENKEN

Mersin University, Turkey

hmenken@mersin.edu.tr

Coauthor: K. R. Mamedov

We consider the second-order differential operator $l(y) = y'' + q(x)y$ with the periodic boundary conditions $y(0) = y(1)$, $y'(0) = y'(1)$ or with the anti-periodic boundary conditions $y(0) = -y(1)$, $y'(0) = -y'(1)$. We assume that $q(x)$ is a complex-valued function in the space $C^4[0, 1]$ satisfying $\int_0^1 q(x) dx = \int_0^1 q^2(x) dx = 0$. The present work deals with the basis property in $L_p(0, 1)$ ($1 < p < \infty$) of the root functions of the operator l .

Theorem. The system of eigenfunctions of the periodic or the anti-periodic boundary value problem above forms a basis in $L_p(0, 1)$ ($1 < p < \infty$).

On the multisublinear operators

LAHCÈNE MEZRAG

M'sila University, Algeria

lmezrag@yahoo.fr

Let X be a Banach space and Y be a Banach lattice. Let $T : X \rightarrow Y$ be a sublinear operator, namely, subadditive and positively homogeneous. We denote by ∇T the set of all linear operators $u : X \rightarrow Y$ such that $u(x) \leq T(x)$ for all x in X . We know that ∇T is not empty if Y is a complete Banach lattice, and $T(x) = \sup\{u(x) : u \in \nabla T\}$, moreover, the supremum is attained. If Y is simply a Banach lattice then ∇T is empty in general. In this communication we introduce and study the category of multisublinear operators and their relation to multilinear operators. We try to find a relation between multisublinear and multilinear like the above equality. In the case of sublinear operators, to have $T(x) = \sup\{u(x) : u \in \nabla T\}$, we use the Hahn-Banach theorem. For multilinear operators, there is no Hahn-Banach theorem. But there are several positive answers for particular multisublinear operators. We continue by introducing to this category the concept of Cohen strongly p -summing operators and we prove the property of *Pietsch's domination theorem*. We end by discussing some relations between multilinear and multisublinear operators.

OPUC from the point of view of block Jacobi matrices theory and related difference-differential lattices

OLEKSII MOKHONKO

National Shevchenko University, Kyiv, Ukraine

`alexeymohonko@univ.kiev.ua`

The theory of orthogonal polynomials on the unit circle (OPUC) has experienced a splash of activity lately thanks to primarily Simon's disquisition. In 2003 five-diagonal structure of the multiplication operator by the independent variable in $L^2(\mathbb{T}, d\rho)$ was first discovered. Then Berezhansky and Dudkin obtained the same result as a simple corollary of a general construction. It was shown that this operator (unitary in $L^2(\mathbb{T}, d\rho)$ and normal in $L^2(\mathbb{C}, d\rho)$) has three-diagonal block structure: it has a block Jacobi matrix. In 2006 Golinskii built an example of a difference-differential lattice on the unit circle \mathbb{T} similar to the Toda chain on the real line \mathbb{R} .

Orthogonal polynomials become components of generalized eigenvectors of the unitary operator of multiplication by the independent variable with block Jacobi matrices in $L^2(\mathbb{T}, d\rho)$. Here ρ is a spectral measure of the original unitary operator that was mapped into the operator of multiplication by the use of the projective spectral theorem. The next step is to show the meaning of Verblunsky coefficients in view of block Jacobi matrix theory, because a lot of OPUC results are formulated in terms of these coefficients.

Presented talk will be devoted to several classical OPUC results that can be nicely interpreted from the point of view of block Jacobi matrix theory (Szegő recursion and some others). The corresponding relations between difference-differential lattices in OPUC theory (e.g. Schur flow equations), analogous matrix flows and the corresponding operator differential equations (generalized Lax equations) will be presented.

The probabilistic modular spaces

KOUROSH NOUROUZI

Toosi University of Technology, Tehran, Iran

`nourouzi@kntu.ac.ir`

Let X be a linear space. A modular ρ is a nonnegative valued function defined on X satisfying the following conditions:

- (M1) $\rho(x) = 0$ if and only if $x = 0$,
- (M2) $\rho(ax) = \rho(x)$ provided $|a| = 1$,
- (M3) $\rho(ax + by) \leq \rho(x) + \rho(y)$ provided $a, b \geq 0$ and $a + b = 1$,
- (M4) $\rho(a_n x) \rightarrow 0$ provided $a_n \rightarrow 0$.

In this talk, a generalization of the concept of modular spaces is given. In this way, instead of nonnegative numbers, to any point a distribution function is assigned. We also investigate some properties of probabilistic modular spaces.

Existence conditions for a global strong solution to a class of nonlinear evolution equations in a Hilbert space

MUKHTARBAY OTELBAEV

Eurasian National University, Astana, Kazakhstan

erj@mail.ru

Coauthor: Y. N. Seitkulov

Let H be a separable real Hilbert space, and $p \in (1, \infty)$, $\theta \in (-\infty, +\infty)$, $a \in (0, \infty)$. Let $C^\infty(H; 0, a)$ be the set of infinitely smooth functions on $[0, a]$ with values in H . The completion of $C^\infty(H; 0, a)$ in the norm $\|f\|_{H_{p,\theta}[0,a]}^p = \int_0^a |A^\theta f(\eta)|_H^p d\eta$ is denoted $H_{p,\theta}[0, a]$. We consider the Cauchy problem

$$u'_t + Au + B(u, u) = f(t), \quad u(0) = 0, \quad 0 < t < a, \quad (1)$$

where A is a self-adjoint nonnegative operator with a completely continuous inverse, $B(u, g)$ is a bilinear operator, and $f \in H_{p,\theta}[0, a]$. Problem (1) is said to be globally strongly solvable if the condition $f \in H_{p,\theta}[0, a]$ implies that problem (1) has a solution $u(t)$ in $(0, a)$ such that $u' + Au \in H_{p,\theta}[0, a]$. Assume that B is subordinate to A , that is,

$$|(A + E)^{-\gamma} B(u, g)| \leq c [|(A + E)^{\gamma_0} u| |(A + E)^{\gamma_0 + \frac{1}{2}} g| + |(A + E)^{\gamma_0} g| |(A + E)^{\gamma_0 + \frac{1}{2}} u|]$$

for all γ and γ_0 that satisfy $\gamma_0 = \delta_0 - \frac{\gamma}{2}$, $-\infty < \gamma \leq \frac{3}{4}$, and $\delta_0 > 0$. Let $\langle B(u, g), g \rangle = 0$.

Theorem. Assume also $0 \leq \delta_0 - \frac{\theta}{2} < \frac{1}{2}$, $(\delta_0 - \frac{\theta}{2})p' < 1$, $\frac{1}{p} + \frac{1}{p'} = 1$. Problem (1) is globally strongly solvable in $H_{p,\theta}[0, a]$ if and only if for all $a > 0$ the system of equations

$$\begin{cases} u'_t + Au + B(u, u) = f(t), & u(0) = 0, & (f \in H_{p,\theta}), \\ -g' + Ag + B_u^* g = 0, & g(a) = 0, \\ g(t) = |A^\theta f|_H^{p-2} A^{2\theta} f, & (0 < t < a), \end{cases}$$

has only the trivial solution, where B_w^* is the adjoint of B_w , and $B_w g = B(w, g) + B(g, w)$.

First-order multiplicative differential equations and their applications

ALİ ÖZYAPICI

Ege University, İzmir, Turkey

emine.kurpinar@ege.edu.tr

Coauthor: Emine Kurpinar

A new kind of calculus, multiplicative calculus, is developed and the definition of a new kind of derivative and integral are given within this calculus. From this point, it is natural to study multiplicative differential equations involving multiplicative derivatives of various orders. The idea of getting alternative mathematical formulation from different perspectives of many problems in science and engineering give rise to multiplicative differential equations. Indeed, new techniques can be developed to set up mathematical models and indicate many problems in science and engineering covering multiplicative differential equations. Additionally, some problems can be expressed in an easier way by using multiplicative differential equations. In this paper, multiplicative differential equations are clarified and some first-order multiplicative differential equations are studied with their applications.

An application of extremal points in dual spaces

SHAHRAM REZAPOUR

Azerbaijan University of Tarbiat Moallem, Tabriz, Iran

sh.rezapour@azaruniv.edu

By using extremal points of dual spaces, we shall improve an incorrect result of the well-known book of Ivan Singer [*Best Approximation in Normed Linear Spaces by Elements of Linear Subspaces*, Springer, 1970].

The inclusion theorem for Cohen strongly multilinear operators

KHALIL SAADI

Université de M'sila, Algeria

kh_saadi@yahoo.fr

In this communication, we give the relation between the class of Cohen strongly p -summing and multiple p -summing multilinear operators. We establish an inclusion between these classes which is a natural generalization of a result due to J. S. Cohen (1973). We also prove a multilinear generalization of an important characterization of strongly p -summing in the linear case, namely, the result stating that the space of conjugates of strongly p -summing is characterized as a Banach space of absolutely p^* -summing operators. This linear result goes back to Cohen, who proved it by using the fact that the bidual of an absolutely p -summing operator is also absolutely p -summing. We end this communication by studying the concept of strongly p -summing multilinear operators introduced by Verónica Dimant (2003), we compare this class with the class of Cohen strongly p -summing of which we prove that the space of Cohen strongly p -summing is included in that of strongly p^* -summing, where the space Y^* is a $\mathcal{L}_{p,\lambda}$ -space.

Rheological effects on the flow of two immiscible fluids in a vertical porous circular cylinder

NIRMAL C. SACHETI

Sultan Qaboos University, Al Khod, Oman

nirmal@squ.edu.om

Coauthor: P. Chandran

Steady flow of two incompressible immiscible fluids in an infinite vertical circular cylinder surrounded by a co-axial thin porous medium has been considered. The flow field in the free fluid region of the cylinder comprises a non-Newtonian bubble and an adjoining Newtonian fluid. The flow in the bounding porous region has been modelled using generalised Darcy's law. This leads to a system of coupled ordinary differential equations with associated boundary and matching conditions. We have obtained exact analytical solutions of these equations in each of the three flow domains. The solutions—expressed in terms of modified Bessel functions—have been analysed to bring out the effects of the permeability of the porous medium, viscous film thickness and the non-Newtonian features of the bubble on the velocities and pressure gradient. The flow configuration considered herein is an idealisation of a number of practically important applications, such as flow of emulsions through narrow capillaries, multi-phase flow through porous media and deformation of red blood cells in a capillary system.

Application of quasiasymptotic boundedness of distribution on wavelet transform

KATERINA SANEVA

Cyril and Methodius University, Skopje, Macedonia

saneva@feit.ukim.edu.mk

Coauthors: A. Bučkovska & S. Pilipović

In this paper we investigate the dependence of the localization properties of the distributional wavelet transform $\mathcal{W}_g f(b, a)$ on the localization of the analyzing distribution $f \in \mathcal{S}'(R)$. Contrary to the approaches based on classical estimations, we apply the theory of asymptotic behavior of distributions to the analysis of the asymptotic behavior of the wavelet transform. Assuming that the distribution $f \in \mathcal{S}'(R)$ is quasiasymptotically bounded at 0 or infinity (resp. $b_0 \in R$) with respect to some regularly varying function, we obtain novel results for the localization of its wavelet transform $\mathcal{W}_g f(b, a)$ (resp. $\mathcal{W}_g f(b_0, a)$).

Some remarks on nonlinear spaces and nonlinear equations

KAMAL N. SOLTANOV

Hacettepe University, Ankara, Turkey

soltanov@hacettepe.edu.tr

In this work we consider some class of the nonlinear spaces which can be pn - or qn -spaces and study their properties. In particular, we investigate their connection with Lebesgue and Sobolev spaces, prove the embedding of one such space to another or to Sobolev space, furthermore studying the compactness of such inclusions. Spaces of such type were investigated earlier by the author, but the spaces considered here are essentially different from previous. It should be noted that each of the spaces considered here is corresponded to a certain nonlinear mapping being its domain of definition as usual. Further, using the investigated spaces, we consider a nonlinear problem and study its solvability and the behaviour of its solutions.

Strong ideal convergence in probabilistic metric spaces

CELALETTİN ŞENÇİMEN

Süleyman Demirel University, Isparta, Turkey

sencimen@fef.sdu.edu.tr

Coauthor: S. Pehlivan

In this work we introduce the concepts of strongly ideal convergent sequence and strong ideal Cauchy sequence in a probabilistic metric (PM) space endowed with the strong topology, and show that every strongly ideal convergent sequence in a PM space is strong ideal Cauchy. Next, we define the strong ideal limit points and the strong ideal cluster points of a sequence in a PM space and prove that a strong ideal limit point is also a strong ideal cluster point, and that the set of all strong ideal cluster points of a sequence in a PM space is strongly closed.

Contour integrals associated with differential equations

TANFER TANRIVERDI

Harran University, Şanlıurfa, Turkey

ttanriverdi@harran.edu.tr

For any n , the contour integral

$$y = \cosh^{n+1} x \oint_C \frac{\cosh(zs)}{(\sinh z - \sinh x)^{n+1}} dz$$

is associated with the differential equation $y'' + (\lambda + n(n+1) \operatorname{sech}^2(x)) y = 0$. With the same procedure, for any n , the contour integral

$$y = \sinh^{n+1} x \oint_C \frac{\sinh(zs)}{(\cosh z - \cosh x)^{n+1}} dz$$

is associated with the differential equation $y'' + (\lambda - n(n+1) \operatorname{csch}^2(x)) y = 0$. Solutions of both differential equations for $n = 2$ are obtained by calculating residues.

On a problem for parabolic-hyperbolic type equation with non-smooth line of type change

AHMADJON K. URINOV

Fergana State University, Uzbekistan

urinovak@mail.ru

Coauthor: I. U. Khaydarov

In the present paper the uniqueness of a boundary value problem for a parabolic-hyperbolic equation with non-smooth line of type change is proved. Let Ω be the plane domain bounded in the first quadrant by $x = 1$, $y = 1$, in the fourth quadrant by $x - y = 1$, $x = 0$, and in the second quadrant by $y = 0$, $y - x = 1$. We set $\Omega_0 = \Omega \cap \{x > 0, y > 0\}$, $\Omega_1 = (\Omega \setminus \bar{\Omega}_0) \setminus \{x + y = 0\}$, $OA^* = \{x = 0, -1 < y < 0\}$, $OB^* = \{y = 0, -1 < x < 0\}$. Consider the parabolic-hyperbolic equation

$$0 = L_\lambda u \equiv \begin{cases} u_{xx} - u_y - \lambda_1^2 u, & (x, y) \in \Omega_0, \\ u_{xx} \operatorname{sign} y + u_{yy} \operatorname{sign} x - \lambda_2^2 u \operatorname{sign}(x + y), & (x, y) \in \Omega_1, \end{cases}$$

in Ω , where λ_1 and λ_2 are given complex numbers.

Problem F. Find a function $u(x, y)$ such that

- (1) u is a regular solution of the equation $L_\lambda u = 0$ in $\Omega \setminus \{xy(x+y) = 0\}$;
- (2) $u(x, y) \in C(\bar{\Omega}) \cap C^1[\Omega \cup OA^* \cup OB^* \setminus \{x+y=0\}]$;
- (3) $u(1, y) = \varphi(y)$ for $0 \leq y \leq 1$, $u(0, y) = f_1(y)$ for $-1 \leq y \leq 0$, $u(x, 0) = g_1(x)$ for $-1 \leq x \leq 0$, $u_x(0, y) - u_x(0, -y) = f_2(y)$ for $-1 < y < 0$, and $u_y(x, 0) - u_y(-x, 0) = g_2(x)$ for $-1 < x < 0$, where φ, f_j, g_j ($j = 1, 2$) are given functions.

It is proved that if $\operatorname{Re} \lambda_1^2 \geq |\operatorname{Im} \lambda_2|^2 + 1/2$, $\varphi \in C[0, 1]$, $f_1, g_1 \in C[0, 1] \cap C^2(0, 1)$, $f_2, g_2 \in C^1(0, 1)$, and $f_1(0) = g_1(0)$, then Problem F has a unique solution.

Index

Abdullayev, 78

Abdullayev, 54

Abreu-Blaya, 16

Achour, 96

Agranovich, 20

Aida-zade, 48

Aida-zade, 52

Aitouche, 102

Akça, 88

Akgün, 5

Akhalaia, 35

Akın, 64

Akperov, 78

Aksoy, 31

Aktaş, 64

Aktymbayeva, 59

Alessandrini, 57

Allan, 96

Almeida, 78

Andersson, 27

Aripov, 96

Arizmendi, 10

Ashyralyev, 87

Ashyralyev, 88, 89, 91, 92

Ateş, B., 93

Ateş, Ü. D., 97

Aubakirov, 62

Aulaskari, 5

Avcı, 31

Aydın, 101

Ayele, 78

Baideldinov, 79

Balakayeva, 59

Barsegian, 1

Bartosiewicz, 97

Barza, 21

Bastos, 6

Bastos, 29

Begehr, 32

Belhout, 9

Benabderahmanne, 97

Benaissa, 40

Benseridi, 99

Benzine, 9

Berdyshev, 87

Bhattacharyya, 98

Bitew, 78

Blazsak, 95

Bolat, 64

Bory-Reyes, 16

Boukrouche, 48

Bounit, 6

Bourgeat, 1

Boussetila, 49

Boussetila, 55

Bouzar, 68, 69

Boykov, 98

Brackx, 13

Bressan, 41

Bučkovska, 107

Buchukuri, 90

Bulboacă, 20

Bulut, 21

Burenkov, 79

Burenkov, 83, 86

Butzer, 2

Carrillo, 10

Catană, 68

Cerejeiras, 13

Chaili, 68

Chandran, 99

Chandran, 106

Chang, 61

Chattopadhyay, 98

Chinchaladze, 32

Chkadua, 90

Cho, 59

Choi, 40

Choi, 42

Colombo, 49

Coriasco, 73

Covachev, 88

Çakmak, 67

Çelebi, 31, 97, 102

Çevik, 6

Dadashova, 24

Dali, 69

Dalla Riva, 21

Dasgupta, 69

De Knock, 13

Del Santo, 41

Demir, 50

Demirci, 88

Derks, 34

De Schepper, 13

Diblík, 64, 65

Dilmi, 99
Di Teodoro, 37
 Di Teodoro, 32
Doniavi, 27, 28
 Dovbush, 6
 Drihem, 79
 Drygaś, 27
Dubatovskaya, 29

Ebrahimi, 27
Edmunds, 83
 Ekincioglu, 80
El-Mennaoui, 6
 Erdem, 50
 Erdoğan, 88
 Eriksson, 14
 Eskandarzade, 27, 28
 Evans, 2

 Fardigola, 33
Fernandes, 6
Fisher, 60
 Fonte, 41
Frederico, 97
 Fujita, 76

 Gaiko, 99
 Gaissinski, 28, 100
 Garello, 69
 Garetto, 69
 Gauthier, 7
 Gentili, 14
 Georgiev, 41
Georgieva-Celakoska, 26
 Gerçek, 88
 Gezgin, 89
 Ghisa, 21
 Gil, 70
 Giorgadze, 33
Giorgashvili, 90
 Gishlarkaev, 100
Glaeske, 61
 Gogatishvili, 80
 Golberg, 22
 Golubeva, 93
 Goncharov, 22, 80
Gourdin, 70
Gök, 58
 Gramchev, 70
Gramchev, 72
 Gramsch, 3
 Guliyev, S. Z., 51
 Guliyev, V. S., 7, 81

Guliyev, V. S., 80, 81
 Guseinov, 65
 Gül, 7
Gürses, 95
 Güven, 8

 Habibullin, 94
 Hajji, 100
 Hamidov, 51
 Hammond, 8
 Handzel, 52
 Hasanov, A., 52
Hasanov, A., 57
 Hasanov, J. J., 81
 Hashemiparast, 52
Héreau, 4
 Hirose, 42
Hitrik, 4
Hörmann, 69
Hung, 18
 Huseynov, 82

Iskakova, 53
Iskenderov, 51
 Israfilov, 22
Israfilov, 5, 11
 Iwasaki, 70

 Jachmann, 42
 Jaiani, 33
Jain, 86
 Johnsen, 70
 Jolevska-Tuneska, 60
 Jung, 42
Jung, 40

 Kähler, 14
 Kabanikhin, 53
 Kalantarov, 43
Kalmenov, B. T., 101
 Kalmenov, T. S., 53
Kalmenov, T. S., 101
 Kanguzhin, 101
 Kaptanoğlu, 8, 8
 Karapınar, 101
 Karasözen, 101
 Karimov, 34
Karlovich, 6
 Karupu, 22
Kasumov, 56
 Kats, 23
 Kaya, 102
 Kaymakçalan, 65

Kerchouche, 102
Khaydarov, 108
Khoroshko, 48
 Kilbas, 60, 61
 Kim, B. J., 61
 Kim, B. S., 61
Kim, B. S., 61
 Kinoshita, 43
 Kokilashvili, 23, 82
Kokilashvili, 8, 83, 84
 Koplatadze, 65
 Kordnourie, 102
 Kordulová, 34
 Koroleva, 62
Korshunova, 52
Kovacheva, 88
 Köksal, 89
 Krainer, 71
 Krasnov, 14
 Krump, 15
Kúdelčíková, 64
Kurpinar, 105
 Kussainova, 82

Lakshmanan, 94
 Lamberti, 83
Lamberti, 79
 Lanza de Cristoforis, 24
Lanza de Cristoforis, 21
 Laskri, 9
 Laville, 15
Leutwiler, 14
 Liu, 71
Lopes, 29
 Lukashov, 89
Luna-Elizarrarás, 15

 Maad, 34
 Mabrouk, 102
 Macías-Cedeño, 15, 24
Makatsaria, 35
Makhmutov, 5
 Mamedkhanov, 24
Mamedov, 103
 Manhas, 9
 Manjavidze, 35
 Maremonti, 43
Martínez, 92
 Masiha, 103
 Matsuura, 76
 Matsuyama, 44
Melrose, 72
 Menken, 103

Merouani, 99
 Mert, 66
 Meskhi, 83
 Mezrag, 103
 Mochizuki, 44
Mohamad, 88
 Mohammed, 35, 71
 Mokhonko, 104
Moorhouse, 8
 Moreno-García, 16
Mostafaei, 102
 Moussai, 9
 Mukanova, 53

 Nakano, 89
 Nakazawa, 45
Narazaki, 45
 Natroshvili, 90, 90
 Nishihara, 45
Nouiri, 97
 Nourouzi, 104
 Nursultanov, 62

 Oberguggenberger, 72
Oberguggenberger, 69
Oinarov, 79
 Oktay, 25
Orunkhanov, 53
 Otáhalová, 77
 Otelbaev, 105

 Özbekler, 66
Özçağ, 60
 Özdemir, 91
 Özfidan, 84
 Öztop, 9
Öztürk, B. A., 58
 Öztürk, E., 91
Özüğür, 58
 Özyapıcı, 105

 Paatashvili, 84
Paatashvili, 23
 Palacios, 10
 Pandey, 94
 Paneah, 54, 66
Pasha, 102
Pashaev, 93
 Pastor, 92
Pehlivan, 107
 Pekcan, 94
 Peña-Peña, 16
 Perotti, 16

Petrov, 52
Pilipović, 72
Pilipović, 107
Piskarev, 92
Plaksa, 25
Polat, 45
Prykarpatsky, 95

Qi, 10
Qian, 17
Qiao, 17

Rafeiro, 84
Ragimov, 28
Rahimov, 54
Rajabov, 35
Rajabova, 36
Rakhmatullaeva, 87
Rakisheva, 54
Rebbani, 55
Rebbani, 49
Reissig, 46
Reissig, 47
Reséndis, 10
Resnick, 55
Rezapour, 106
Ridha, 36
Robbins, 8
Rochon, 72
Rogosin, 29
Rovenski, 28, 100
Ruzhansky, 46, 73
Ruzhansky, 44, 74
Růžičková, 65

Saadi, 106
Sabadini, 17
Sacheti, 106
Sacheti, 99
Safarov, 84
Saigo, 61
Saitō, 74
Saitoh, 77
Saks, 37
Samko, 85
Samko, 82
Sandstede, 34
Saneva, 107
Santos, 29
Sasane, 11
Saspaeva, 58
Schulze, 73
Seitkulov, 105

Sergienko, 56
Shapiro, 15, 24
Sharapudinov, 85
Sharifov, F. A., 56
Sharifov, Y. A., 56
Silindir, 95
Sincich, 57
Sjöstrand, 4
Šmid, 17
Soltanov, 107
Sommen, 18
Sommen, 16
Son, 18
Sözen, 92
Stens, 2
Stolk, 4
Struppa, 14
Sugimoto, 46, 73
Szablikowski, 95

Şençimen, 107
Şerbetçi, 80, 81

Tamrazov, 25
Tanrıverdi, 108
Tararykova, 86
Tasmambetov, 63
Tatar, 57
Taylan, 58
Tedjani, 97
Tezel, 58
Tiryaki, 64, 67
Todorova, 47
Toft, 73
Topal, 67
Torres, 97
Tovar, 11, 18
Tozman, 11
Tuncay, 67
Tuneski, 26
Turunen, 74
Tülü, 8

Uesaka, 46
Uğur, 58
Uğurlu, 47
Umeda, 74
Urinov, 108
Usmanov, 37

Ünal, 67
Vaitekhovich, 30
Van, 18

Vanegas, 37
 Vasilieva, 26
Venkov, 41
Ventsel, 98
 Vlacci, 19
Vũ Ngọk, 4

 Wakabayashi, 74
 Wang, 37
 Weber, 58
 Wikström, 12
 Wirth, 47
 Wong, 75
Wong, 35, 69, 71
Wu, 7, 12

 Xu, W., 12
 Xu, Y., 38, 58

 Yakubov, 86
 Yantır, 67
 Yıldırım, 4
 Yoo, 63
 Yordanov, 47
 Yüksel, 38

 Zafer, 67
Zafer, 66
Zazashvili, 90
 Zelinskii, 26
Zenati, 48
 Zhang, Z., 19
Zhao, 12
 Zhdanov, 38
 Zheltukhin, 95
 Zhilisbaeva, 58
 Zhong, 39
 Zorboska, 12
Zouyed, 55