Name: Grade: /10

Math 102, Calculus II, Spring 2024, Sec. 3 & 13, HTK
Quiz 5, Tue., Apr. 16

1. Let $f(x,y) = \dfrac{y\cos(x-\pi/2)}{2x^2+y^2}$ if $(x,y) \neq (0,0)$, and $f(0,0)=0$. Is f continuous at $(0,0)$? Is f differentiable at $(0,0)$? Explain.

2. Find the equation of the tangent plane to the surface $z = g(x^2-y^2, e^{xy})$ at $(x,y)=(1,0)$ given that $g(1,1)=4$, $g_1(1,1)=5$, $g_2(1,1)=6$, and g is differentiable.

1. Let $f(x,y) = \dfrac{y\ln(1+x)}{2x^2+y^2}$ if $(x,y) \neq (0,0)$, and $f(0,0)=0$. Is f continuous at $(0,0)$? Is f differentiable at $(0,0)$? Explain.

2. Find the equation of the tangent plane to the surface $z = g(x^2+2y^2, 3\sin(xy))$ at $(x,y)=(0,1)$ given that $g(2,0)=7$, $g_1(2,0)=8$, $g_2(2,0)=9$, and g is differentiable.

1-) $\displaystyle\lim_{\substack{(x,y)\to(0,0)\\ y=x}} f(x,y) = \lim_{x\to 0} \frac{x\cos(x-\pi/2)}{3x^{\cancel{2}}} = \lim_{x\to 0}\frac{\sin x}{3x} = \frac{1}{3},$

$\displaystyle\lim_{\substack{(x,y)\to(0,0)\\ y=3x}} f(x,y) = \lim_{x\to 0} \frac{3x\cos(x-\pi/2)}{11x^{\cancel{2}}} = \lim_{x\to 0}\frac{3\sin x}{11x} = \frac{3}{11}.$

Since $\frac{1}{3} \neq \frac{3}{11}$, by the two-path test, $\displaystyle\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist.
So f is not continuous at $(0,0)$. Then f is not differentiable at $(0,0)$.

2-) Put $u = x^2-y^2$, $v = e^{xy}$. Then $z = g(u,v)$, $g_1 = g_u$, $g_2 = g_v$, and $(u,v)=(1,1)$ when $(x,y)=(1,0)$.

$\displaystyle\frac{\partial z}{\partial x} = \frac{\partial g}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial g}{\partial v}\frac{\partial v}{\partial x} = g_1 2x + g_2 y e^{xy},$

$\displaystyle\frac{\partial z}{\partial y} = \frac{\partial g}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial g}{\partial v}\frac{\partial v}{\partial y} = g_1(-2y) + g_2 x e^{xy},$

$\displaystyle\frac{\partial z}{\partial x}(1,0) = g_1(1,1)\,2(1) + g_2(1,1)\,0\,e^0 = 10,$

$\displaystyle\frac{\partial z}{\partial y}(1,0) = g_1(1,1)(-2)0 + g_2(1,1)\,1e^0 = 6.$

$z-4 = 10(x-1) + 6(y-0)$ is the equation of tangent plane.

1-) $\lim\limits_{\substack{(x,y)\to(0,0)\\ y=x}} f(x,y) = \lim\limits_{x\to 0} \frac{\cancel{x}\ln(1+x)}{3x^{\cancel{2}}} = \lim\limits_{x\to 0} \frac{\frac{1}{1+x}}{3} = \frac{1}{3}$,

$\lim\limits_{\substack{(x,y)\to(0,0)\\ y=3x}} f(x,y) = \lim\limits_{x\to 0} \frac{3x\ln(1+x)}{11x^{2}} = \lim\limits_{x\to 0} \frac{\frac{3}{1+x}}{11} = \frac{3}{11}$.

Since $\frac{1}{3} \neq \frac{3}{11}$, by the two-path test, $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ does not exist.

So f is not continuous at $(0,0)$. Then f is not differentiable at $(0,0)$.

2-) Put $u = x^2 + 2y^2$, $v = 3\sin(xy)$. Then $z = g(u,v)$, $g_1 = g_u$, $g_2 = g_v$,

and $(u,v) = (2,0)$ when $(x,y) = (0,1)$.

$\frac{\partial z}{\partial x} = \frac{\partial g}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial g}{\partial v}\frac{\partial v}{\partial x} = g_1\, 2x + g_2\, 3y\cos(xy)$,

$\frac{\partial z}{\partial y} = \frac{\partial g}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial g}{\partial v}\frac{\partial v}{\partial y} = g_1\, 4y + g_2\, 3x\cos(xy)$,

$\frac{\partial z}{\partial x}(0,1) = g_1(2,0)\,2(0) + g_2(2,0)\,3(1)\cos 0 = 27$,

$\frac{\partial z}{\partial y}(0,1) = g_1(2,0)\,4(1) + g_2(2,0)\,3(0)\cos 0 = 32$.

$z - 7 = 27(x-0) + 32(y-1)$ is the equation of tangent plane.