Name: Grade: /10

Math 102, Calculus II, Spring 2024, Sec. 3 & 13, HTK
Quiz 4, Tue., Mar. 26

1. Find the equation of the plane that contains the points P_1 $(1, 2, 3)$ and P_2 $(3, 2, 1)$ and that is parallel to the line $\mathbf{r} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k} + t(3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k})$.

1. Find the equation of the plane that contains the points P_1 $(3, 4, 5)$ and P_2 $(5, 4, 3)$ and that is parallel to the line $\mathbf{r} = \alpha\mathbf{i} + \beta\mathbf{j} + \gamma\mathbf{k} + t(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$.

1. Find the equation of the plane that contains the points P_1 $(3, 5, 7)$ and P_2 $(7, 5, 3)$ and that is parallel to the line $\mathbf{r} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k} + t(-1\mathbf{i} + 2\mathbf{j} + 5\mathbf{k})$.

1. Find the equation of the plane that contains the points P_1 $(-1, 2, 5)$ and P_2 $(5, 2, -1)$ and that is parallel to the line $\mathbf{r} = \alpha\mathbf{i} + \beta\mathbf{j} + \gamma\mathbf{k} + t(3\mathbf{i} + 5\mathbf{j} + 7\mathbf{k})$.

2. Find the length of the curve $\mathbf{r}(t) = e^t \cos t\,\mathbf{i} + e^t \sin t\,\mathbf{j} + 8\mathbf{k}$, $0 \le t \le 1$.

2. Find the length of the curve $\mathbf{r}(t) = t\cos t\,\mathbf{i} + t\sin t\,\mathbf{j} + \frac{2\sqrt{2}}{3}t^{3/2}\mathbf{k}$, $0 \le t \le 1$.

1-) $\vec{u} = \overrightarrow{P_1P_2} = 2\vec{i} - 2\vec{k}$ is on plane and $v = 3\vec{i} + 4\vec{j} + 5\vec{k}$ is parallel to plane.

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 0 & -2 \\ 3 & 4 & 5 \end{vmatrix} = -6\vec{j} + 8\vec{k} + 8\vec{i} - 10\vec{j} = 8\vec{i} - 16\vec{j} + 8\vec{k}.$$

$\vec{n} = \vec{i} - 2\vec{j} + \vec{k}$ is normal to plane. $x - 1 - 2(y-2) + z - 3 = 0.$

1-) $\vec{u} = \overrightarrow{P_1P_2} = 2\vec{i} - 2\vec{k}$ is on plane and $\vec{v} = \vec{i} + 2\vec{j} + 3\vec{k}$ is parallel to plane.

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 0 & -2 \\ 1 & 2 & 3 \end{vmatrix} = -2\vec{j} + 4\vec{k} + 4\vec{i} - 6\vec{j} = 4\vec{i} - 8\vec{j} + 4\vec{k}.$$

$\vec{n} = \vec{i} - 2\vec{j} + \vec{k}$ is normal to plane. $x - 3 - 2(y-4) + z - 5 = 0.$

1-) $\vec{u} = \overrightarrow{P_1P_2} = 4\vec{i} - 4\vec{k}$ is on plane and $\vec{v} = -\vec{i} + 2\vec{j} + 5\vec{k}$ is parallel to plane.

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 0 & -4 \\ -1 & 2 & 5 \end{vmatrix} = 4\vec{j} + 8\vec{k} + 8\vec{i} - 20\vec{j} = 8\vec{i} - 16\vec{j} + 8\vec{k}.$$

$\vec{n} = \vec{i} - 2\vec{j} + \vec{k}$ is normal to plane. $x - 3 - 2(y-5) + z - 7 = 0.$

1-) $\vec{u} = \overrightarrow{P_1P_2} = 6\vec{i} - 6\vec{k}$ is on plane and $\vec{v} = 3\vec{i} + 5\vec{j} + 7\vec{k}$ is parallel to plane.

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 6 & 0 & -6 \\ 3 & 5 & 7 \end{vmatrix} = -18\vec{j} + 30\vec{k} + 30\vec{i} - 42\vec{j} = 30\vec{i} - 60\vec{j} + 30\vec{k}.$$

$\vec{n} = \vec{i} - 2\vec{j} + \vec{k}$ is normal to plane. $x + 1 - 2(y-2) + z - 5 = 0.$

2-) $\vec{r}'(t) = (e^t\cos t - e^t\sin t)\vec{i} + (e^t\sin t + e^t\cos t)\vec{j}$.

$|r'(t)| = \sqrt{e^{2t}\cos^2 t \cancel{- 2e^{2t}\cos t\sin t} + e^{2t}\sin^2 t + e^{2t}\sin^2 t \cancel{- 2e^{2t}\sin t\cos t} + e^{2t}\cos^2 t}$

$= \sqrt{2e^{2t}} = \sqrt{2}\, e^t$.

Length $= \int_0^1 \sqrt{2}\, e^t = \sqrt{2}\, e^t \Big|_0^1 = \sqrt{2}(e-1)$.

2-) $\vec{r}'(t) = (\cos t - t\sin t)\vec{i} + (\sin t + t\cos t)\vec{j} + \sqrt{2}\, t^{1/2}\vec{k}$.

$|\vec{r}'(t)| = \sqrt{\cos^2 t \cancel{- 2t\cos t\sin t} + t^2\sin^2 t + \sin^2 t \cancel{+ 2t\sin t\cos t} + t^2\cos^2 t + 2t}$

$= \sqrt{1 + t^2 + 2t} = t+1$ since $0 \le t \le 1$.

Length $= \int_0^1 (t+1)\,dt = \frac{t^2}{2} + t \Big|_0^1 = \frac{3}{2}$.