Name: Grade: /10

Math 102, Calculus II, Spring 2024, Sec. 3 & 13, HTK
Quiz 2, Thu. & Fri., Feb. 29 & Mar. 1
Show all your work and name any tests you use.

1. It is given that the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2+1}$ is convergent. How many terms of the series must be added to estimate the sum of the series with an error of less than 0.01 ?

It is given that the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+3\sqrt{n})^3}$ is convergent. How many terms of the series must be added to estimate the sum of the series with an error of less than 0.001 ?

2. Determine all values of x at which the series $\sum_{n=0}^{\infty} (n+1)^2 \left(\frac{x}{x+2}\right)^n$ converges absolutely, converges conditionally, or diverges.

Determine all values of x at which the series $\sum_{n=1}^{\infty} \frac{1}{n}\left(1+\frac{1}{x}\right)^n$ converges absolutely, converges conditionally, or diverges.

1-) Both series are alternating. If we estimate their sums s by their Nth partial sums s_N, the errors are less than the absolute values of the first omitted terms, $|a_{N+1}|$.

For $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2+1}$, $|\text{error}| < \left|\frac{(-1)^{N+2}}{(N+1)^2+1}\right| = \frac{1}{(N+1)^2+1} < 0.01$,

$(N+1)^2+1 > 100$, $(N+1)^2 > 99$, $N+1 \geq 10$, $N \geq 9$.

So 9 terms are enough.

For $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+3\sqrt{n})^3}$, $|\text{error}| < \left|\frac{(-1)^{N+2}}{(N+1+3\sqrt{N+1})^3}\right| = \frac{1}{(N+1+3\sqrt{N+1})^3} < 0.001$,

$(N+1+3\sqrt{N+1})^3 > 1000$, $N+1+3\sqrt{N+1} > 10$, $N+1 \geq 5$, $N \geq 4$.

So 4 terms are enough.

2-) We use ratio test for absolute convergence and divergence.

For $\sum_{n=0}^{\infty} (n+1)^2 \left(\frac{x}{x+2}\right)^n$, $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{(n+2)^2}{(n+1)^2} \left|\frac{x}{x+2}\right|^{n+1} \left|\frac{x+2}{x}\right|^n =$

$= \left|\frac{x}{x+2}\right| \lim_{n\to\infty} \frac{n^2+4n+4}{n^2+2n+1} = \left|\frac{x}{x+2}\right| < 1$, $|x| < |x+2|$ for absolute convergence.

So $\text{dist}(x,0) < \text{dist}(x,-2)$; then $x > -1$ is where series converges absolutely.

If $\left|\frac{x}{x+2}\right| > 1$, $|x| > |x+2|$, $x < -1$, the series diverges. When $x = -1$, the series is $\sum_{n=0}^{\infty}(n+1)^2(-1)^n$ which diverges by nTT since $\lim_{n\to\infty} a_n \neq 0$.

So the series diverges for $x \leq -1$ and there is no x at which the series converges conditionally.

For $\sum_{n=1}^{\infty}\frac{1}{n}\left(1+\frac{1}{x}\right)^n$, $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\frac{n}{n+1}\left|1+\frac{1}{x}\right|^{n+1}\left|\frac{1}{1+\frac{1}{x}}\right|^n =$

$= \left|1+\frac{1}{x}\right| < 1$, $\left|\frac{x+1}{x}\right| < 1$, $|x+1| < |x|$ for absolute convergence.

So $\text{dist}(x,-1) < \text{dist}(x,0)$; then $x < -\frac{1}{2}$ is where series converges absolutely.

If $\left|1+\frac{1}{x}\right| > 1$, $|x+1| > |x|$, $x > -\frac{1}{2}$, the series diverges. When $x = -\frac{1}{2}$, the series is $\sum_{n=1}^{\infty}\frac{1}{n}(-1)^n$. This series converges since it is alternating harmonic. It does not converge absolutely since $\sum_{n=1}^{\infty}\frac{1}{n}$ diverges since this is harmonic. So the given series converges conditionally at $x = -\frac{1}{2}$.