Topological aspects of ideal conduction

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**Topological invariants**

- Drude weight as equilibrium susceptibility
  - Hamiltonian: \( W(\Phi) = \sum_{i} \frac{|\phi_{i} + \Phi|^{2}}{2m} + \frac{\Phi}{2m} \)
  - \( \Phi \) represents perturbation (electric field)
  - Current: \( J(\Phi) = \frac{\partial E_{\Phi}(\Phi)}{\partial \Phi} \)
  - Drude weight: \( D = \frac{1}{N} \sum_{i} \frac{\partial E_{\Phi}(\Phi)}{\partial \Phi} \)

- Relation to other ideal transport quantities
  - Drude weight: \( D \)
  - Meissner weight: \( D^{*} \)
  - Superfluid weight: \( D^{**} \)

- Drude weight as a topological invariant
  - Similar to the modern theory of polarization (King-Smith & Vanderbilt: PRB 1993; Resta: RMP 1994)

- Drude weight as a topological invariant
  - Topological invariants
  - Quantization of transport related quantities
  - Edge currents at interfaces

- Quantum Hall effect & TKNN invariant
  - Kane-Mele model: edge states
  - Hamiltonian (Kane and Mele, PRL 2005):

- **Quantum systems**
  - Characterized by topological invariant
  - Quantization of transport related quantities
  - Edge currents at interfaces (Halperin, PRB 1984)
  - Quantum Hall systems, topological insulators

- Berry phase & Berry curvature
  - Adiabatic cycle:
  - Continuous limit:
  - Drude weight:
  - Ideal conduction:

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- **Kane-Mele model: edge states**
  - Hamiltonian (Kane and Mele, PRL 2005):

- **Drude weight/ideal conduction**
  - Derived in a seminal paper by Kohn (PR, 1964) as the criterion to distinguish conductors from insulators
  - Related to many-body localization
  - Measure of ideal (non-diffusive) conduction:

- **References:**

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**Edge states (cont’d)**

- Band structure of system with TB/CDW interface

- Current/resolved Drude weight show similar behavior

- Transport coefficients and ODLRO
  - How can one distinguish transport coefficients \( D_{p} \), \( D^{*} \), and \( D^{**} \)?
  - Definition of \( p \)-current based on generator of \( p \)-particle translations:

- Quantization
  - Flux quantization inside a cavity of a superconductor (Deaver and Fairbank, PRL 1961; Byers and Yang, PRL 1961)

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