Experimental background

Experimental setup

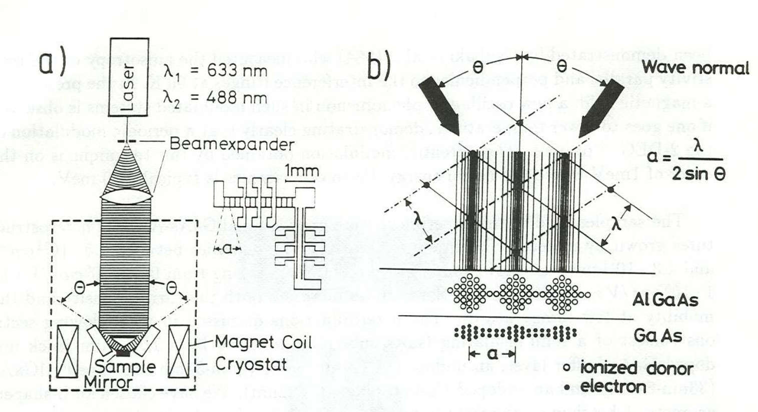
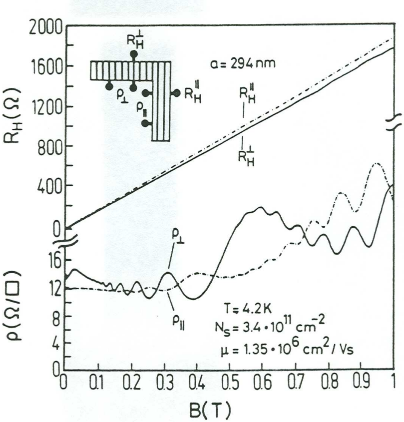


Figure: (a) The L-shaped sample is illuminated with an interference pattern of laser light. (b) Near the intensity maxima many donors are ionized, near the minima only a few. This leads to a periodic modulation potential acting on and a periodic density modulation of the 2DES. [D. Weiss, K. von Klitzing, K. Ploog, and G. Weimann, in *High Magnetic Fields in Semiconductor Physics II*, Vol. 87 of *Springer Series in Solid-State Sciences*, edited by G. Landwehr (Springer-Verlag, Berlin, 1989), p. 357; Europhys. Lett. **8**, 179 (1989)]

Typical results



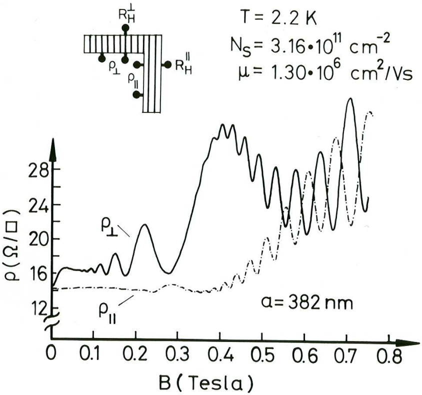


Figure: The longitudinal resistance $\rho_\perp$ measured for current flow across the equipotentials shows a strong positive low-field magnetoresistance followed by a magnetic field regime with pronounced new oscillations, which at high magnetic fields are superimposed by the Shubnikov – de Haas oscillations. The longitudinal resistance $\rho_{||}$ measured for current flow along the equipotentials shows weaker oscillation of opposite phase and no magnetoresistance effect at low fields. The Hall resistances show no characteristic effects, but indicate a slight difference of electron density in both legs of the L-shaped sample, consistent with the slightly different periods of the SdH oscillations.

Summary: Commensurability (Weiss) oscillations

Longitudinal resistance for current flow perpendicular to equipotentials, $\rho_\perp$, shows

- strong positive magnetoresistance at very low B- fields
- in an intermediate B regime new type of oscillations, with minima at

$$2R_c = a\left(\lambda - \frac{1}{4}\right), \quad (\lambda = 1,\ 2,\ 3, \dots).$$

  The positions of these minima are equidistant on a $1/B$ scale and depend on the modulation period a and, via cyclotron radius $R_c = v_F/\omega_c = \ell^2 k_F$, on the electron density $n_{\mathrm{el}} = k_F^2/2\pi$.

- SdH oscillations at larger B values, with minima at

$$\nu = 2\pi\ell^2 \equiv (\ell k_F)^2 = 2n, \quad (n == 1,\ 2,\ 3, \dots).$$

  The positions of these minima are also equidistant in $1/B$ and density dependent.

Longitudinal resistance for current flow parallel to equipotentials, $\rho_\parallel$, shows

- no magnetoresistance
- weak antiphase commensurability oscillations
- usual SdH oscillations

The Hall resistance shows no significant effect.

Basic ideas about the origin of Weiss oscillations

- The modulation potential $V(x)$ lifts the degeneracy of Landau levels and leads to a periodic dependence of $E_n(X)$ on the center coordinate X.
- This will lead to current-carrying states in y-direction:

$$\langle \phi_{X,n} | \hat{v}_y | \phi_{X,n} \rangle = \frac{\partial E_n(X)}{\partial \hbar k_y} \neq 0$$

- The magnitude of this drift motion will oscillate with the position of the center of the state, if this does not average out the modulation (extent $\sim n \cdot a$).
- This drift in y-direction should affect σ_{yy}, i.e., ρ_{xx}.

For weak modulation a perturbative treatment of the modulation potential may be sufficient.

Quantum treatment of 1D modulation

We consider a weak electric modulation with potential energy $V(x) = V_0 \cos Kx$. To first order with respect to the small parameter $V_0/(\hbar\omega_c)$, energy eigenvalues and eigenstates of the modulated system are given

$$E_n(X) = \varepsilon_n + \langle X, n|V(x)|X, n\rangle = \varepsilon_n + U_n \cos KX \,,$$
$$|\phi_{X,n}\rangle = |X, n\rangle + \sum_{n'\neq n} |X, n'\rangle \frac{V_{n'n}(X)}{\varepsilon_n - \varepsilon_{n'}} \,,$$

where $\varepsilon_n = \hbar\omega_0(n + 1/2)$ and $|X, n\rangle$ are energy eigenvalue and eigenstate of the unmodulated Landau system, $U_n = V_0 \mathcal{J}_{nn}(\ell^2 K^2/2)$ is the dispersive energy correction due to the modulation, and

$$V_{n'n}(X) = \langle X, n'|V(\hat{x})|X, n\rangle = V_{n'n}^{(1)} e^{iKX} + V_{n'n}^{(-1)} e^{-iKX} \,,$$

with $V_{n'n}^{(\lambda)} = \delta_{|\lambda|,1}(i\lambda)^{|n-n'|}(V_0/2)\mathcal{J}_{n'n}([\ell K]^2/2)$, are the matrix elements of the modulation potential in the Landau representation. The

$$\mathcal{J}_{n'n}(Q) = \frac{\tilde{n}!}{\sqrt{n'!n!}} Q^{\frac{1}{2}|n'-n|} e^{-\frac{1}{2}Q} L_{\tilde{n}}^{(|n'-n|)}(Q)$$

have been defined before. Note that $U_n \cos KX = V_{nn}(X)$.

B-dependence of modulated energy spectra

$$E_n(X) = \varepsilon_n + U_n \cos KX\,, \quad \varepsilon_n = \hbar\omega_c(n+1/2)\,.$$

With $R_n = \ell\sqrt{2n+1}$ and $Q = \ell^2 K^2/2$ one finds for $KR_n \gg 1$

$$U_n = V_0\, e^{-Q/2} L_n(Q) \approx V_0\, J_0(KR_n) \approx V_0\, \frac{\cos(KR_n - \pi/4)}{\sqrt{\pi KR_n/2}}\,.$$

This is already a good approximation for $KR_n \geq 3$.

The width of the modulation broadened Landau energy bands will shrink to zero for $KR_n - \pi/4 = (2\lambda - 1)\pi/2$, which for $K = 2\pi/a$ yields

$$2R_n = a\big(\lambda - \frac{1}{4}\big)\,, \quad (\lambda = 1,\ 2,\ 3,\ \ldots)\,.$$

In the modulated system the energy eigenstates will carry current in y-direction:

$$v_y(n;X) = \langle \phi_{X,n} | \hat{v}_y | \phi_{X,n} \rangle = \frac{-1}{m\omega_c}\,\frac{dE_n(X)}{dX} = \frac{U_n}{m\omega_c}\,\sin KX\,.$$

Typical energy spectrum

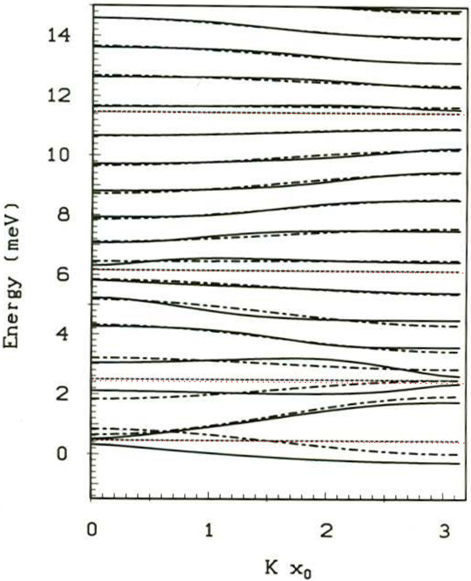


Figure: Typical energy spectrum for weak electrostatic modulation at fixed B ($\hbar\omega_c = 0.5\,\mathrm{meV}$). Solid lines: exact numerical calculation; dash-dotted: first order perturbation expansion. The red dashed lines indicate the "flat-band" situations $U_n = 0$.

DOS is modulated 2DES

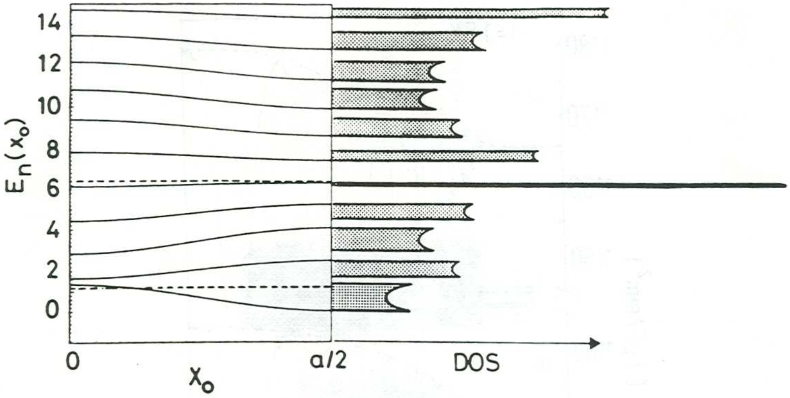


Figure: Typical energy spectrum of modulated system at fixed B, and resulting DOS

First calculations

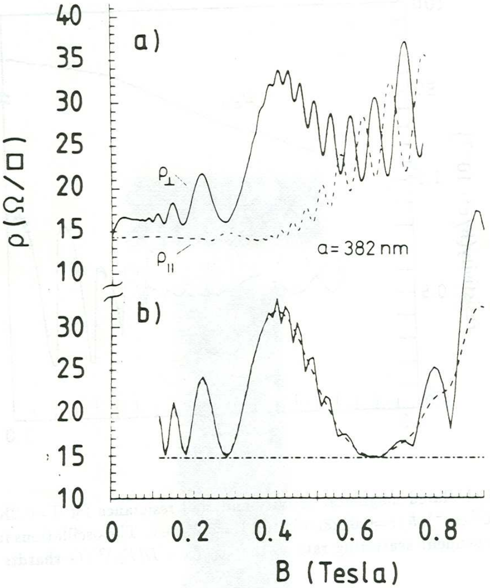
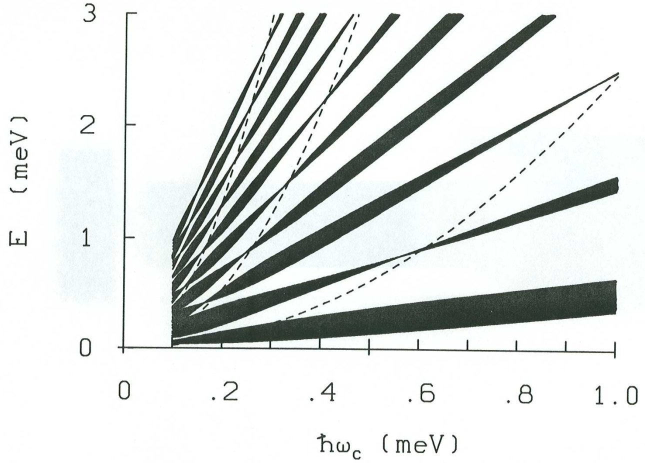


Figure: Experiment (a) and calculation (b) on the basis of Kubo formula, but in the approximation of constant relaxation time. The Weiss oscillations of $\rho_\perp$ are nicely reproduced, those of $\rho_{||}$ not at all. The calculated SdH oscillations are not reliable. The low B regime is not covered, since the calculation is done in the Landau representation considering only a finite number of Landau levels. [after R.R. Gerhardts, D. Weiss, and K. v.Klitzing, Phys. Rev. Lett. **62**, 1173 (1989)]. A similar experiment on samples with etched surfaces and a corresponding calculation were presented at the same time by R.W. Winkler, J.P. Kotthaus, and K. Ploog, Phys. Rev. Lett. **62**, 1177 (1989).

DOS effect

The calculation in the constant relaxation time approximation yields the Drude result. To obtain the "antiphase oscillations", one should calculated the relaxation rate similar to the SdH case. Landau fan for modulated 2DES:



Scattering and band conductivity

The modulation potential $V(x)$ introduces as a qualitatively new phenomenon current carrying energy eigenstates $|n, X)$, $H|n, X) = E_n(X)|n, X)$, with a finite group velocity in y direction,

$$(n, X|v_y|n, X) = -\frac{1}{m\omega_c}\frac{dE_n(X)}{dX},$$

while $(n, X|v_x|n, X) = 0$ still holds, In high-mobility systems, this leads to a "band conductivity" contribution to σ_{yy} that increases with decreasing impurity-scattering rate (just as the conductivity in the absence of a magnetic field), whereas the usual diffusive magnetoconductivity ("scattering conductivity") is non-zero only because of impurity scattering and decreases with decreasing scattering rate.
For a rough estimate, we may neglect vertex corrections to obtain

$$\sigma_{yy}(E) \sim \frac{\pi e^2\hbar}{\mathcal{F}}\,\mathrm{Tr}\; v_y A(E) v_y A(E).$$

A typical contribution to the scattering conductivity is of the form

$$|(n, X|v_y|n \pm 1, X)|^2 A_n(E; X) A_{n\pm 1}(E; X)$$

with $E_n \approx E = E_F$, the Fermi energy and the estimate
$A_{n\pm 1}(E \approx E_n; X) \approx \Gamma_{n\pm 1}/[2\pi(\hbar\omega_c)^2]$.

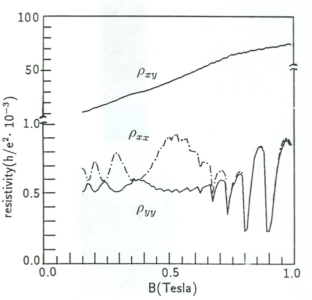
Calculations neglecting vertex corrections

The corresponding contribution to the band conductivity is

$$\left[(n,X|v_y|n,X)A_n(E_F;X)\right]^2.$$

Since $\left[A_n(E_F;X)\right]^2 \sim A_n(E_F;X)2/[\pi\Gamma_n]$, the product of the spectral function factors is now by a factor $4(\hbar\omega_c)^2/(\Gamma_n\Gamma_{n\pm1}) \sim 4(\omega_c\tau)^2$ larger.

In early calculations we have neglected the vertex corrections and calculated the scattering conductivity from the off-diagonal elements of the velocity operator (which are to lowest order in the modulation the same as in the Landau representation), and the band conductivity from the diagonal components.



After C. Zhang and R. R. Gerhardts, Phys. Rev. B **41**, 12850 (1990).

Commensurability oscillations at higher temperatures

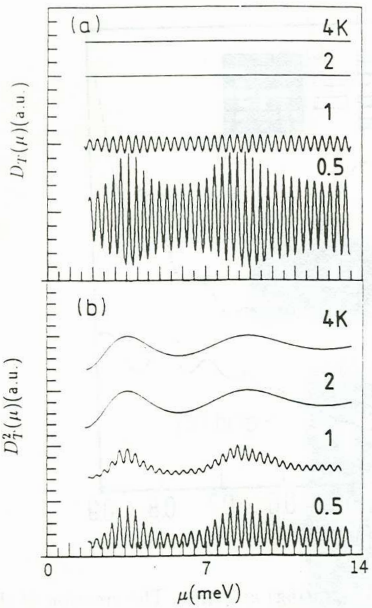


Figure: The Weiss oscillations of the scattering conductivity are proportional to the square of the spectral functions (of the DOS), and thus survive at higher temperatures than the commensurability oscillations of the TDOS.

Magnetocapacitance of modulated DES

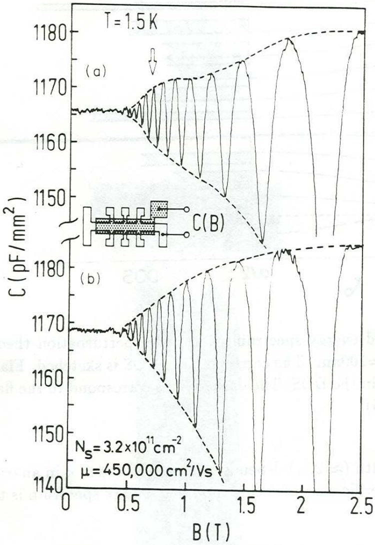


Figure: Magnetocapacitance of a 2DES with (a) and without (b) a 1D lateral superlattice. Note that the commensurabilty oscillations appear as amplitude modulation of the SdH oscillations, and disappear with the latter at low B values, in contrast to the conductivity oscillations.

Magnetotransport for finite-range scatterers

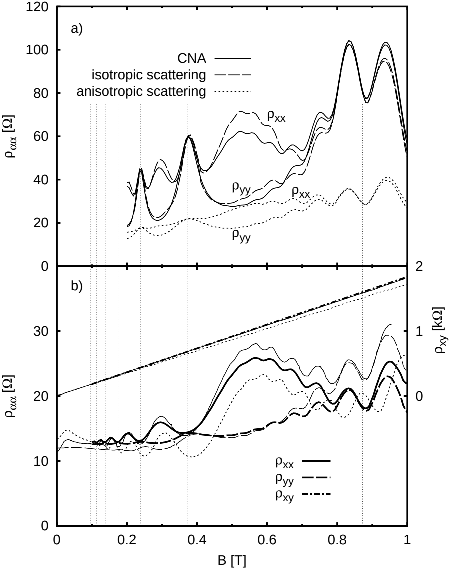


Figure: Effect of scattering anisotropy. (a) Resistivities for isotropic scattering in c-number approximation for the selfenergy and in HDA (Green function diagonal in H-representation) both with $\Gamma_0 = 1.85\,\mu$eV, and for anisotropic scattering in HDA with $r_0 = 15$ nm and $\Gamma_0 = 92.5\,\mu$eV, yielding all $\rho_0 = 12.3\,\Omega$. (b) Thick lines: calculated resistivities for Coulomb scattering, with $z = 14$ nm, $\Gamma_0 = 0.8$ meV, i.e. $\rho_0 = 12\,\Omega$. Remaining parameters in (a) and (b): $T = 4.2$ K, $n_{\rm el} = 3.4 \times 10^{15}\,\mathrm{m}^{-2}$, $V_0 = 0.45$ meV, $a = 294$ nm. The vertical lines indicate the minima of the Weiss oscillations (flat-band conditions). Thin lines: experimental results after D. Weiss *et al.* (1989).

[After J. Groß and R.R. Gerhardts, Phys. Rev. B (2002)]