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Dept:................

Date: April 28, 2015
Time: 17:45-19:25
Instructor: Dilek Güvenç

MATH 260 MIDTERM EXAM II

IMPORTANT

- Check that there are 4 questions in your booklet.
- Show all your work. Correct results without sufficient explanation and correct notation may not get full credit. Please, be neat.
- Write your name on each page of your booklet.
- Do NOT use your mobile phone as a calculator, turn it off during the exam.
- Do NOT forget to return the tables and the "cheat-sheet", as well.

1	2	3	4	TOTAL
25	20	30	25	100

********* **GOOD LUCK!** *********

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1. Independent random samples from two normal populations produced the following results:

Sample 1	1.2	3.1	1.7	2.8	3
Sample 2	4.2	2.7	3.6	3.9	

a) Test the equality of the two population variances at significance level of 1%. **(10 Points)**

b) Can we conclude that mean of the second population is greater than the mean of the first one? Use the conclusion in part (a) and test at $\alpha = 0.05$. **(15 Points)**

$n_1 = 5 \quad \Sigma X = 11.8 \quad \Sigma X^2 = 30.78 \quad \bar{X} = 2.36 \quad s_1^2 = \left(30.78 - \frac{(11.8)^2}{5}\right)/4 = 0.733$

$n_2 = 4 \quad \Sigma Y = 14.4 \quad \Sigma Y^2 = 53.1 \quad \bar{Y} = 3.6 \quad s_2^2 = \left(53.1 - \frac{(14.4)^2}{4}\right)/3 = 0.42$

a) $H_0: \sigma_1^2 = \sigma_2^2 \quad F_{cal} = \frac{s_1^2}{s_2^2} = 1.745 \quad F_{0,01}(4,3) = 28.7$

$H_A: \sigma_1^2 > \sigma_2^2$

Since $F_{cal} = 1.745 \not> F_{0,01}(4,3) = 28.7$ do not reject H_0, conclude that $\sigma_1^2 = \sigma_2^2$ at $\alpha = 0.01$

b) $H_0: \mu_2 = \mu_1$ or $H_0: \mu_2 - \mu_1 = 0$ by part (a) we assume that $\sigma_1^2 = \sigma_2^2$

$H_A: \mu_2 > \mu_1$ $\quad H_A: \mu_2 - \mu_1 > 0$

$\Rightarrow S_p^2 = \frac{(5-1)s_1^2 + (4-1)s_2^2}{5+4-2} = 0.599 \Rightarrow S_p = 0.77 \quad t_{0,05}(7) = 1.895$

$T_{cal} = \frac{3.6 - 2.36 - 0}{0.77\sqrt{\frac{1}{5}+\frac{1}{4}}} = 2.4$ Since $T_{cal} = 2.4 > t_{0,05}(7) = 1.895$ we reject H_0, conclude that $\mu_2 > \mu_1$ at $\alpha = 0.05$.

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2. It is desired to test $H_0 : \mu = 50$ against $H_A : \mu < 50$ using $\alpha = 0.10$. Distribution of the population is uniform with standard deviation 20. A random sample of size 64 will be drawn from this population.

a) What is the (approximate) sampling distribution of $\overline{X}$ if H_0 is true? **(4 Points)**

b) What is the (approximate) sampling distribution of $\overline{X}$ if H_A is true and population mean is 45? **(3 Points)**

c) If $\mu = 45$ what is the probability that the test would lead the researcher to commit a Type II error? **(10 Points)**

d) What is the power of this test for detecting the alternative when $\mu = 45$? **(3 Points)**

$H_0: \mu = 50$ $\quad n = 64$ $\quad \sigma = 20$

$H_A: \mu < 50$ $\quad \alpha = 0{,}10$

a) By C.L.T $\overline{X} \overset{app.}{\sim} N\left(\mu_{\overline{X}} = 50,\ \sigma_{\overline{X}} = \frac{20}{\sqrt{64}} = 2{,}5\right)$

b) If $\mu_A = 45$ by C.L.T $\overline{X} \overset{app.}{\sim} N\left(\mu_{\overline{X}} = 45,\ \sigma_{\overline{X}} = 2{,}5\right)$

c) $\beta = P(\text{Type II Error}) = P(\text{do not reject } H_0 \mid H_A \text{ is true})$

We reject H_0 if $\quad \dfrac{\overline{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} \leq -Z_{0,10}$ $\qquad Z_{0,1} = 1{,}28$

$\overline{X} \leq 50 - (1{,}28)\left(\frac{20}{\sqrt{64}}\right) = 46{,}8$

We do not reject if $\overline{X} > 46{,}8$

$\Rightarrow \beta = P(\overline{X} > 46{,}8 \mid \mu = 45) = P\left(Z > \frac{46.8 - 45}{2.5}\right) = P(Z > 0.72)$

$= 0{,}5 - A(0.72) = 0{,}5 - 0{,}2642 = 0{,}2358$

d) Power of the test $= 1 - \beta = 1 - 0{,}2358 = 0{,}7642$

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3. In a consumer privacy survey 328 internet users indicated their level of agreement with the following statement: "The government needs to be able to scan internet messages and user communications to prevent fraud and other crimes." The number of users in each response category is summarized as follows:

Agree strongly	Agree somewhat	Disagree somewhat	Disagree strongly	Total
59	108	82	79	328

a) Write the null and alternative hypotheses you would use to determine whether the opinions of internet users are evenly divided among the 4 categories. **(4 Points)**

b) Test the hypotheses in part (a) at significance level of 5%. **(10 points)**

c) Can we conclude that majority of internet users agree (both strong or somewhat) that the government needs to be able to scan internet messages and user communications to prevent fraud and other crimes? Test at $\alpha = 0.10$. Find P-value of the test. **(16 Points)**

a) $H_0: p_1 = p_2 = p_3 = p_4 = \frac{1}{4}$

H_A: There is at least one inequality

b) Under H_0 $E(X_i) = 82$. $d.o.f = 3$ $\chi^2_{0,05}(3) = 7,81$

$$\chi^2_{cal} = \frac{(59-82)^2}{82} + \frac{(108-82)^2}{82} + 0 + \frac{(79-82)^2}{82} = 14.8$$

Since $\chi^2_{cal} = 14.8 > \chi^2_{0,05}(3) = 7.81$ we reject H_0, conclude that opinions of internet users are not evenly divided.

c) $H_0: p_A \leq 0.50$ $\hat{p}_A = \frac{59+108}{328} = 0,51$

$H_A: p_A > 0.50$

$$Z_{cal} = \frac{0,51 - 0,50}{\sqrt{\frac{(0.5)(0.5)}{328}}} = \frac{0.01}{0,0276} = 0.36 \qquad Z_{0,1} = 1.28$$

Since $Z_{cal} = 0,36 \not> Z_{0,1} = 1,28$ we do not reject H_0.
Conclude that majority do not agree at $\alpha = 0.10$.

$P\text{-}Value = P(Z > 0.36) = 0,5 - A(0.36) = 0,5 - 0,1406 =$
$= 0,3594$

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4. Suppose a fire insurance company wants to relate the amount of fire damage in major residential to the distance between burning house and the nearest fire station. A sample of 15 recent fires in the city where study was conducted, is selected. The amount of damage, Y (in thousands of dollars) and the distance between fire and the nearest fire station, x (in miles) are recorded for each fire.

Some calculations: $S_{xx} = 34.789$, $S_{XY} = 171.25$, $S_{YY} = 911.517$, $\sum x = 49.2$, $\sum Y = 396.2$
Use these calculations to answer the following questions:

a) Find the fitted line. **(5 Points)**

b) Predict the damage when the distance is 3 miles. **(3 Points)**

c) Estimate the standard deviation of Y values when the distance is 3 miles. **(3 Points)**

d) Repeat part (c) when the distance is 5.5 miles. **(2 Points)**

e) Do the data contradict the claim that over the range of x values covered in the study, the increase in damage, for per mile increase in the distance is more than 4-thousand dollars? Test at $\alpha = 0.05$ **(12 Points)**

a) $\hat{\beta}_1 = \frac{S_{XY}}{S_{XX}} = \frac{171.25}{34.789} = 4.92$ $\quad \bar{X} = \frac{\Sigma X}{n} = \frac{49.2}{15} = 3.28 \quad n = 15$

$\bar{Y} = \frac{\Sigma Y}{n} = \frac{396.2}{15} = 26.41 \Rightarrow \hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X} = 26.41 - (4.92)(3.28)$

$\hat{\beta}_0 = 10.27$

$\Rightarrow$ Fitted line, $\hat{Y} = 10.27 + 4.92X$

b) If $X = 3$ $\quad \hat{Y} = 10.27 + (4.92)(3) = 25.032 \Rightarrow 25032$ dollars

c) when $X = 3$ $\quad$ Std. dev (Y) = std dev $(\varepsilon) = s$

$s^2 = \frac{SSE}{n-2}$ $\quad SSE = S_{YY} - \frac{S_{XY}^2}{S_{XX}} = 911.517 - \frac{(171.25)^2}{34.789} = 68.533$

$\Rightarrow s^2 = \frac{68.533}{13} = 5.27 \Rightarrow s = 2.296$

d) $s = 2.296$ (constant variance/std dev for given x)

e) $H_0: \beta_1 \leq 4$
$H_A: \beta_1 > 4$

Under H_0 $\quad T_{cal} = \frac{\hat{\beta}_1 - \beta_1}{\frac{s}{\sqrt{S_{XX}}}} = \frac{4.92 - 4}{\frac{2.296}{\sqrt{34.789}}} = \frac{0.92}{0.3893}$

$= 2.363$; $\quad$ d.o.f $= 15 - 2 = 13$

$t_{0.05}^{(13)} = 1.771$

Since $T_{cal} = 2.363 > t_{0.05}^{(13)} = 1.771$ we reject H_0, conclude that increase in damage, for per mile increase in distance is more than 4-thousand dollars.