MATH102: QUIZ VIII

Name:
Student Number:
Section: 4
Date: April 15, 2009

Question:
Find and classify the critical points of $f(x,y) = x^2y^2 - 2x^2 + x^3 - 4y^2$.

Solution:
a) Critical Points: $f_x = 2xy^2 - 4x + 3x^2 = 0, \quad f_y = 2x^2y - 8y = 0$ From these equations one finds four points $(0,0), (4/3,0), (-2,\sqrt{5}), (-2,-\sqrt{5})$
b) Classification:

$$f_{xx} = 2y^2 - 4 + 6x, \quad f_{yy} = 2x^2 - 8, \quad f_{xy} = 4xy \tag{1}$$

Let

$$\Delta(x,y) = f_{xx}f_{yy} - (f_{xy})^2$$

Then
i) $(0,0)$: $\Delta(0,0) > 0, f_{xx}(0,0) < 0$ Then $(0,0)$ is a local maximum point
ii) $(4/3,0)$: $\Delta(0,0) < 0$, Then $(3/4,0)$ is a local saddle point.
iii) $(-2,\sqrt{5})$: $\Delta(-2,\sqrt{5}) < 0$, Then $(-2,\sqrt{5})$ is a local saddle point.
iv) $(-2,-\sqrt{5})$: $\Delta(-2,-\sqrt{5}) < 0$, Then $(-2,-\sqrt{5})$ is a local saddle point.