MATH102: QUIZ V

Name:
Student Number:
Section: 4
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Question: The following limit exists.

$$\lim_{x\to 0}\left(\frac{2-x\sin x-2\cos x}{x^n}\right)$$

Using the method of series expansion
a) Find all possible values of the positive integer n ($n=1,2,\cdots$)
b) Find the values of the above limit for all possible n values.

SOLUTION: Using the Maclaurin series expansion of $\sin x$ and $\cos x$ we obtain

$$\lim_{x\to 0}\left(\frac{2-x\sin x-2\cos x}{x^n}\right)=$$

$$\lim_{x\to 0}\left(\frac{2-(x^2-x^4/3!+x^6/5!-\cdots)-(2-2x^2/2!+2x^4/4!-2x^6/6!+\cdots)}{x^n}\right)=$$

$$\lim_{x\to 0}\left(\frac{(1/6-1/12)x^4-(1/5!-2/6!)x^6+\cdots)}{x^n}\right)$$

For $n=1,2,3$ the limit is zero but for $n=4$

$$\lim_{x\to 0}\left(\frac{2-x\sin x-2\cos x}{x^4}\right)=1/6-1/12=1/12$$