MATH102: QUIZ IV

Name:
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Section: 4
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Question: **a)** Find the radius and interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x+2)^n}{n}.$$

What values of x does the series converge **b)** absolutely, **c)** conditionally?

SOLUTION: The nth root test (You can also use the ratio test):
$|a_n| = \frac{|x+2|^n}{n}$ for all $n = 1, 2, \ldots$. Hence absolute convergence we must have

$$\lim_{n\to\infty} |a_n|^{1/n} = \lim_{n\to\infty} \frac{|x+2|}{n^{1/n}} = |x+2| < 1$$

or $-3 < x < -1$. The radius of convergence $R = 1$. The interval of convergence for the absolute convergence is $-3 < x < -1$. At the end points: $x = -3$ the series becomes

$$\sum_{n=1}^{\infty} \frac{(-1)^n (-1)^n}{n} = \sum_{n=1}^{\infty} \frac{1}{n}$$

which is divergent. At $x = -1$ the series is the harmonic alternating series hence convergent. Then for convergence the interval convergence is $-3 < x \leq -1$.
Summary:
Absolute convergence for all x with $-3 < x < -1$
Conditional convergence at $x = -1$. Series is convergent for $-3 < x \leq -1$