MATH102: QUIZ III

Name:
Student Number:
Section: 4
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Question: Does the series

$$\sum_{n=5}^{\infty}(-1)^n \frac{1}{\ln n},$$

converge absolutely? does it converge? does it conditionally converge? or does it diverge? Prove your statement.

SOLUTION:a) **Absolute convergence**:

$$a_n = (-1)^n \frac{1}{\ln n}, \quad n = 5, 6, \cdots$$

Hence

$$|a_n| = \frac{1}{\ln n}, \quad n = 5, 6, \cdots$$

The series

$$\sum_{n=5}^{\infty} \frac{1}{\ln n},$$

diverges by comparison test. $\frac{1}{\ln n} \geq \frac{1}{n \ln n}$ for all $n = 5, 6, \cdots$. Since $\sum_{n=5}^{\infty} \frac{1}{n \ln n}$ diverges then $\sum_{n=5}^{\infty} \frac{1}{\ln n}$. Hence the given series $\sum_{n=5}^{\infty}(-1)^n \frac{1}{\ln n}$ does not converge absolutely.
b) **Convergence**: The series $\sum_{n=5}^{\infty}(-1)^n \frac{1}{\ln n}$ converges by alternating series test. $u_n = \frac{1}{\ln n}, \quad n = 5, 6, \cdots$. (i).$u_n > 0$ for all $n = 5, 6, \cdots$. (ii).You can prove that $u_{n+1} < u_n$ because $\ln(n+1) > \ln n$ foe all $n = 5, 6, \cdots$. (iii). Finally $\lim_{n\to\infty} u_n = 0$. Hence all three conditions of the alternating series are satisfied. This series does **converge** but **does not converge absolutely**. Hence it **converges conditionally**.