**MATH102: QUIZ II**

**Name:**
**Student Number:**
**Section: 4**
**Date: February 23, 2009**

**Question**
Use a test (Integral or comparison) to show that the following series are convergent or divergent

$$1)\sum_{n=2}^{\infty}\frac{1}{n(\ln n)^3},\quad 2)\sum_{n=1}^{\infty}\frac{2}{1+e^n}$$

**SOLUTIONS**:
1)**Integral Test**. ( The general form of this problem $\sum_{n=2}^{\infty}\frac{1}{n(\ln n)^p}$ was solved in class. We showed that such series converge only when $p>1$. *Only 3 students could solve this problem and 10 students got 0 or 1* (!)) This series converges. **Proof**: Let $f(x)=\frac{1}{x(\ln x)^3}$, for $x\geq 2$. If the improper integral $\int_2^{\infty}f(x)dx$ converges then the given series converges. Hence we evaluate this improper integral.

$$\int_2^{\infty}f(x)dx=\lim_{a\to\infty}\int_2^a\frac{dx}{x(\ln x)^3}$$

letting $\ln x=u$ and using $du=\frac{dx}{x}$ we get

$$\int_2^{\infty}f(x)dx=\lim_{a\to\infty}\int_{\ln 2}^{\ln a}\frac{du}{u^3}=\lim_{a\to\infty}[\frac{1}{2(\ln 2)^2}-\frac{1}{2(\ln a)^2}]=\frac{1}{2(\ln 2)^2}$$

Hence the given series converges by the integral test.

2) This series also converges. We can prove this statement by using both the comparison and integral tests.
(a). **Comparison test**: $\frac{2}{1+e^n}<\frac{2}{e^n}$ for all $n\geq 1$. The series $\sum_1^{\infty}\frac{2}{e^n}=\frac{2}{e-1}$ converges. Hence the series $\sum_{n=1}^{\infty}\frac{2}{1+e^n}<\frac{2}{e-1}$ converges by direct comparison test.
(b). **Integral test**: Let $f(x)=\frac{2}{1+e^x}$ for $x\geq 1$. If the improper integral $\int_1^{\infty}f(x)dx$ converges the the given series converges. Hence

$$\int_1^{\infty}f(x)dx=\lim_{a\to\infty}\int_1^a\frac{2dx}{1+e^x}=\lim_{a\to\infty}\int_1^a\frac{2e^{-x}dx}{1+e^{-x}}$$

Letting $e^{-x} = u$ and using $e^{-x}dx = -du$ we get

$$\int_1^{\infty} f(x)dx = -\lim_{a\to\infty} \int_{e^{-1}}^{e^{-a}} \frac{2du}{1+u} = -2 \lim_{a\to\infty} [\ln(1+e^{-a}) - \ln(1+e^{-1})] = 2\ln(1+e^{-1})$$

Hence the given series converges by the integral test.