Math 102 (Section 4)
Quiz.1
February 18, 2009

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QUESTION: a)Let a sequence be given as $\{1, -\frac{1}{3}, \frac{1}{9}, \cdots, (-1)^n \frac{1}{3^n}, \cdots\}$. Write the nth term, a_n, of the sequence and show that $\lim_{n\to\infty} a_n = 0$.
b) Is $\sum_{n=1}^{\infty} a_n$ convergent. If yes find the sum.

SOLUTION:a) 1. In this sequence if you take $a_n = (-1)^n \frac{1}{3^n}$ then $n = 0, 1, 2, 3, \cdots$ but if you take $a_n = (-1)^n - 1 \frac{1}{3^{n-1}}$ then $n = 1, 2, 3, \cdots$.
2. Proof of $\lim_{n\to\infty} a_n = 0$. We have to find a positive number N such that for all given $\epsilon > 0$ we have $|a_n - L| < \epsilon$ for all $n > M$. We start with
$|a_n - L| = |-1)^n \frac{1}{3^n}| = \frac{1}{3^n} < \epsilon$. When $\epsilon \geq 1$ this inequality is valid for all $n \geq 1 (N = 1)$. When $\epsilon < 1$ this inequality is valid for all $n > N$, where $N = \frac{-\ln \epsilon}{\ln 3}$
b) Here you do not have to know any test for convergence. The series is $\sum_{n=1}^{\infty} (-1)^n \frac{1}{3^n} = -1/3 + 1/9 - 1/27 + \cdots$. This is a geometrical series. For convergence we can look at its sequence of partial sums

$$\begin{aligned} s_n &= \sum_{k=1}^{n} (-1)^k \frac{1}{3^k} = -1/3 + 1/9 - 1/27 + \cdots + (-1)^n \frac{1}{3^n} \\ &= -1/3 - 1/3\,(-1/3 + 1/9 - \cdots + (-1)^{n-1}/3^{n-1}) \\ &= -1/3 - 1/3(s_n - (-1)^n \frac{1}{3^n}) \end{aligned}$$

Hence

$$s_n = \frac{1}{4}(-1 + (-1)^n \frac{1}{3^n})$$

Then as $n \to \infty$ and from part a)

$$\lim_{n\to\infty} s_n = -1/4$$

Since the sequence of partial sums is convergent then the series is convergent and sum of the series $\sum_{n=1}^{\infty} a_n = -1/4$