**MATH 544: METHODS OF APPLIED MATHEMATICS II**

*Final Exam:*

*30 May 2008 Friday 9.00-11.00*

SAZ02

**QUESTIONS:**

[30]**1**. Let

$$y(x) = f(x) + \lambda \int_0^1 (x - s)\, y(s) ds$$

where $f(x) = x^a, \quad a \in \mathbb{R}$. Find the values of $\lambda$ and $a$ so that

a) y(x) has a unique solution. Give the solution

b) y(x) has infinitely many solutions. Give these solutions.

c) y(x) has no solution.

[40] **2.(a)** For $T > 0$ let

$$Q_T = \{(x,t) | 0 < x < L,\ 0 < t \leq T\}$$

(this is exactly the region $D$ defined in the lecture), be a rectangle with its bottom and two vertical sides removed. Let

$$\bar{Q}_T = \{(x,t) | 0 \leq x \leq L,\ 0 \leq t \leq T\}.$$

Then $B_T = \bar{Q}_T - Q_T$ consists of the three missing sides of the original rectangle $Q_T$. Suppose that $w(x,t)$ satisfies the differential inequality

$$w_t - a(x,t) w_{xx} - b(x,t) w_x - c(x,t) w < 0 \quad (x,t) \in Q_T$$

where $a(x,t) \geq 0$ and $c(x,t) \leq 0$ in $Q_T$. Then $w(x,t)$ can not achieve a positive local maximum in $Q_T$

**2.(b)** Let $a, b$ and $c$ be as in part (a). Suppose that $u(x,t)$ is continuous on $\bar{Q}_T$ and satisfies

$$u_t - a(x,t) u_{xx} - b(x,t) u_x - c(x,t) u \leq 0, \quad (x,t) \in Q_T, \tag{1}$$

$$u(x,t) \leq 0, \quad (x,t) \in B_T \tag{2}$$

Then $u(x,t) \leq 0$ on $\bar{Q}_T$.

**2. (c)**. Let $a, c$ and $c$ be as in part (a). Suppose that $u(x,t)$ is continuous on $\bar{Q}_T$ and satisfies

$$u_t - a(x,t)u_{xx} - b(x,t)u_x - c(x,t)u = 0$$

in $Q_T$. If $u$ has a positive maximum or minimum, they are assumed on the boundary $B_T$.

[30]**3**. Consider the ordinary differential system

$$\dot{x} = -y + x(x^2 + y^2 - 1), \tag{3}$$

$$\dot{y} = x + y(x^2 + y^2 - 1). \tag{4}$$

Discuss the stability of the equilibrium solutions of this system and show that the periodic path is unstable.