MATH 544: METHODS OF APPLIED MATHEMATICS II

Second Midterm Exam:

28 April 2008 Monday 15.40-17.30

SAZ21

QUESTIONS:

[35]**1**. Solve the problem:

$$u_t - a u_{xx} = F(x,t), \quad 0 < x < L, \ t > 0, \tag{1}$$

$$u_x(0,t) = u_x(L,t) = 0, \quad t \geq 0, \tag{2}$$

$$u(x,0) = f(x), \quad 0 \leq x \leq L. \tag{3}$$

Explain also how you can prove the uniqueness of the solution of this problem.

[35]**2**. Homogeneous wave equation with initial and boundary value problem. Let

$$u_{tt} - c^2 \, u_{xx} = 0,$$

$$u_{x,0} = f(x), \quad u_t(x,0) = g(x), \quad 0 \leq x \leq L$$

$$u(0,t) = u(L,t) = 0, \quad t \geq 0$$

Under what conditions on the data $\{f(x), g(x)\}$ the above problem is well-posed?.

[35]**3**. Let $D \in R^2$ be a rectangular region $0 < x < a, \ 0 < y < b$. Consider the following Dirichlet's problem

$$u_{xx} + u_{yy} = 0, \quad (x,y) \in D \tag{4}$$

$$u(0,y) = 0, \quad u(a,y) = 0, \quad 0 \leq y \leq b \tag{5}$$

$$u(x,0) = f(x), \quad u(x,b) = 0, \quad 0 \leq x \leq a \tag{6}$$

a. Find the formal solution of the problem.

b. Find restrictions of $f(x)$ so that the formal solution is a solution of the problem.

[35]**4**.