MATH 544: METHODS OF APPLIED MATHEMATICS II

First Midterm Exam:

17 March 2008 Monday 15.40-17.30

SAZ21

QUESTIONS: Choose any three of the following four problems

[35]**1**. Determine the natural boundary condition at $x = b$ for the variational problem defined by

$$J(y) = \int_a^b L(x, y, y')\, dx + G(y(b)), \quad y \in \mathbb{C}^2[a, b], \quad y(a) = y_0$$

where G is a given differentiable function. As an application of this problem let $L = (y')^2, y \in [0, 1], y(0) = 1$, $y(1)$ is unspecified and $G = y^2$. Find y extremizing this problem. Classify the extreme points. (Give reasons for your answers)

[35]**2**. Verify the following approximations for large λ.

(i) $\int_0^\infty e^{-\lambda t} \ln(1 + t^2) dt \sim \frac{2!}{\lambda^3} - \frac{1}{2}\frac{4!}{\lambda^5} + \cdots, \quad \lambda >> 1,$
(ii) $\int_0^1 \sqrt{1 + t}\, e^{\lambda(2t - t^2)}\, dt \sim \sqrt{\frac{\pi}{2\lambda}}\, e^{\lambda}. \quad \lambda >> 1,$
(iii) $\int_1^2 \sqrt{3 + t}\, e^{\frac{\lambda}{t+1}} dt \sim \frac{8}{\lambda} e^{\lambda/2}, \quad \lambda >> 1$

[35]**3**. . Use singular perturbation methods to obtain a uniform approximate solution to the following problem. We assume that $0 < \varepsilon << 1$ and $0 < t < 1$.

$$\varepsilon y'' + (t + 1)y' + y = 0, \quad y(0) = 0,\ y(1) = 1$$

[35]**4**. Consider the differential equation $x^2 u_{xx} - y^2 u_{yy} = 0$. Classify the DE and bring it to its canonical form. Discus (Existence and uniqueness of) the initial value problem on non-characteristic and characteristic curves. As an example let the initial curve be $y = x^2$ and $u(x, x^2) = 0, \quad u_y(x, x^2) = 1$,find the solution.