ASSIGNED EXERCISES OF MATH544:
IntegraL Equations set 8
April, 2001

THE INTEGRAL EQUATIONS AND THE GREEN'S FUNCTION

Ronald B Guenther and John W Lee, Partial Differential Equations of Mathematical Physics and Integral Equations
See also Ch.4 of the lecture notes.

1. Show that the Green's function is unique, if it exists. *Hint.* If both $G(x,y)$ and $H(x,y)$ provides the same $u(x)$ given by

$$u(x) = \int_a^b G(x,y)f(y)dy, \tag{1}$$
$$u(x) = \int_a^b H(x,y)f(y)dy, \tag{2}$$

then

$$\int_a^b [G(x,y) - H(x,y)]f(y)dy = 0$$

.

2. Find Green's function of the following problem

$$-k\,u'' + l\,u = 0, \quad u(0) - u'(0) = 0, \quad u(1) = 0.$$

3. Discuss the existence and and the construction of the Green's function of the following problem

$$-[p(x)u']' + q(x)u = f(x), \quad 0 < x < 1, \tag{3}$$
$$u(0) = u(1), \quad u\prime(0) = u'(1) \tag{4}$$

4. Find Green's function for the initial value problem

$$Lu = f(x), \quad u(0) = 0, \quad u'(0) = 0.$$

Show that the solution has the form

$$u(x) = \int_0^x G(x, y) f(y) dy.$$

5. Let $u'' - u = f(x)$. Find Green's function for the boundary conditions $u(x) \to 0$ ax $|x| \to \infty$.