MATH544: Methods of Applied Mathematics
Final Exam
May 26, 2010

PROBLEMS:

1. Find the solution of the heat equation $u_t = u_{xx} + xe^{-t}$ with $u(0,t) = 0$, $u_x(1,t) + u(1,t) = 0$ and $u(x,0) = 0$.

2. Let $u'' = f(x)$, $x \in (a,b)$. Solve this problem by using the method of Green's Function with the boundary conditions to $u(a) - \alpha^2\, u'(a) = 0$ and $u(b) + \beta^2\, u'(b) = 0$. Here α and β are some constants.

3. **a)** Obtain the most general solution of the integral equation

$$y(x) = f(x) + \lambda \int_0^{2\pi} \sin(x+s)\, y(s) ds.$$

Discuss all possible cases. **b)** Obtain solutions when when $f(x) = x$ and $f(x) = 1$.