MATH544: Methods of Applied Mathematics
Second Midterm Exam
April 29, 2010

PROBLEMS:

1. Using the Green's function technique solve the following Dirichlet problem.

$$\nabla^2 u = 0, \quad -\infty < x, y < \infty, \quad z > 0, \tag{1}$$
$$u(x, y, 0) = f(x, y), \quad -\infty < x, y < \infty. \tag{2}$$

where $f(x, y)$ is a continuous function in R^2.

2. Consider

$$\nabla^2 u(r, \theta) = 0, \quad 0 \le r < a, \tag{3}$$
$$\frac{\partial u(a, \theta)}{\partial r} + \alpha\, u(a, \theta) = f(\theta), \tag{4}$$

with f is 2π periodic continuous function and $\alpha > 0$ a constant.
a. Solve formally by separation of variables.
b. Find reasonable conditions on f so that the formal solution is a solution.

3. Solve the following initial value problem. Under what conditions the problem is well-posed? Find reasonable conditions on f.

$$u_t = a u_{xx}, \quad (a > 0), t > 0, \quad x \in [0, L],$$
$$u(0, t) = u(L, t) = 0, \quad t \ge 0,$$
$$u(x, 0) = f(x), \quad x \in [0, L]$$