ASSIGNED EXERCiSES4:
PDE Set 2
February, 2001

PDE Problems
Copson:
See also Ch.4 of the lecture notes.

1. Find the types of the following differential equations, and reduce them to normal form:

$$\begin{aligned}
i \qquad & y^2r - t = 0, & (1)\\
(ii) \qquad & y^2r + 2ys + t = p, & (2)\\
(iii) \qquad & y^2r + t = 0, & (3)\\
(iv) \qquad & r - 2s + 3t - q - u = 0, & (4)\\
(v) \qquad & (1+x^2)^2r - t = 0, & (5)\\
(vi) \qquad & x^2r - 2xys + y^2 + y^2t = 0 & (6)\\
(vii) \qquad & x^2r + 2xys + y^2t = 0. & (7)
\end{aligned}$$

2. Reduce the folloeing equations to normal form:

$$\begin{aligned}
(i) \qquad & u_{xx} + 2u_{yy} + +3u_{zz} + 4u_{tt} + 2u_{xy} + 2u_{xz} + 2u_{xt} + \\
+ \quad & 4u_{yz} + 4u_{yt} + 6u_{xt} = 0, & (8)\\
(ii) \qquad & u_{xx} + u_{zz} + 2u_{xy} + 2u_{xz} + 2u_{xt} + 2u_{zt} = 0, & (9)\\
(iii) \qquad & u_{xx} + u_{xy} + u_{xz} + u_{xt} = 0. & (10)
\end{aligned}$$

3. Prove that $q^2r - 2pqs + p^2t = 0$ has an intemediate integral $p = qf(u)$, where f is an arbitrary function. Hence show that $y + xf(u) = g(u)$, where g is another arbitrary function

4. Transform the equation $x^2u_{xx} - y^2u_{yy} = 0$ into one with characteristic variables. Hence show that $u = f(xy) + xg(\frac{y}{x})$, where f and g are arbitrary functions.

5. Find the characteristics of $r - yt = 0$. reduce this equation to normal form when $y > 0$.

6. Find the characteristics of $(1 + x^2)r - (1 + y^2)t = 0$, and reduce this equations to its normal form.

7. Prove that $x^2r + 2xys + y^2t = 0$ has an intermediate integral

$$px + qy - u + f(\frac{y}{x}) = 0$$

where f is an arbitrary function. Hence show that

$$u = f(\frac{y}{x}) + xg(\frac{y}{x})$$

where g is an arbitrary function.

8. Show that the equation $qr + (uq - p)a = upt$ has an intermediate integral $p + qu = f(u)$, where f is an arbitarary function. Hence solve the equation.