**ASSIGNED EXERCiSES4:**

**PDE Set 2**

*February, 2001*

## PDE Problems

Copson:

See also Ch.4 of the lecture notes.

**1**. Find the types of the following differential equations, and reduce them to normal form:

$$
\begin{aligned}
i \qquad & y^2 r - t = 0, && (1)\\
(ii) \qquad & y^2 r + 2ys + t = p, && (2)\\
(iii) \qquad & y^2 r + t = 0, && (3)\\
(iv) \qquad & r - 2s + 3t - q - u = 0, && (4)\\
(v) \qquad & (1 + x^2)^2 r - t = 0, && (5)\\
(vi) \qquad & x^2 r - 2xys + y^2 + y^2 t = 0 && (6)\\
(vii) \qquad & x^2 r + 2xys + y^2 t = 0. && (7)
\end{aligned}
$$

**2**. Reduce the folloeing equations to normal form:

$$
\begin{aligned}
(i) \qquad & u_{xx} + 2u_{yy} + +3u_{zz} + 4u_{tt} + 2u_{xy} + 2u_{xz} + 2u_{xt} + \\
+ \quad & 4u_{yz} + 4u_{yt} + 6u_{xt} = 0, && (8)\\
(ii) \qquad & u_{xx} + u_{zz} + 2u_{xy} + 2u_{xz} + 2u_{xt} + 2u_{zt} = 0, && (9)\\
(iii) \qquad & u_{xx} + u_{xy} + u_{xz} + u_{xt} = 0. && (10)
\end{aligned}
$$

**3**. Prove that $q^2 r - 2pqs + p^2 t = 0$ has an intemediate integral $p = qf(u)$, where $f$ is an arbitrary function. Hence show that $y + xf(u) = g(u)$, where $g$ is another arbitrary function

**4**. Transform the equation $x^2u_{xx} - y^2u_{yy} = 0$ into one with characteristic variables. Hence show that $u = f(xy) + xg(\frac{y}{x})$ , where $f$ and $g$ are arbitrary functions.

**5**. Find the characteristics of $r - yt = 0$. reduce this equation to normal form when $y > 0$.

**6**. Find the characteristics of $(1 + x^2)r - (1 + y^2)t = 0$, and reduce this equations to its normal form.

**7**. Prove that $x^2r + 2xys + y^2t = 0$ has an intermediate integral

$$px + qy - u + f(\frac{y}{x}) = 0$$

where $f$ is an arbitrary function. Hence show that

$$u = f(\frac{y}{x}) + xg(\frac{y}{x})$$

where $g$ is an arbitrary function.

**8**. Show that the equation $qr + (uq - p)a = upt$ has an intermediate integral $p + qu = f(u)$ , where $f$ is an arbitarary function. Hence solve the equation.