MATH 544: METHODS OF APPLIED MATHEMATICS II
Final Exam
May 5, 2016; 15.30-17.30

QUESTIONS: Choose any three of the following four problems

[35] **1**. An initial value problem is given as follows:

$$\begin{aligned} &u_t = a u_{xx}, \quad (a>0), t>0, \quad x \in [0,1], \\ &u(0,t) = u(1,t) = 0, \quad t \geq 0, \\ &u(x,0) = x - x^2, \quad x \in [0,1] \end{aligned}$$

Is this problem well posed?

[35] **2**. Let a 2-dimensional dynamical system be given by

$$\begin{aligned} \dot{x} &= 2y^2 + 2x - 8y, \\ \dot{y} &= y^2 + x - 2y \end{aligned}$$

Find the equilibrium points and classify them. Discuss also the stability of this system against linear perturbations.

[35] **3**. Let $J(y) = \int_0^L (y'')^2 dx$ with constraint $\int_0^L y^2\, dx = 1$ and with boundary conditions $y(0) = y''(0) = 0, \quad y(L) = y''(L) = 0$

(a). Find the function extremezing the functional J.

(b). Consider the same problem without the constraint.

[35] **4**. Consider the following initial+boundary value problem

$$u_{tt} + a^2\, u_{xxxx} = 0, \quad t>0, \quad 0<x<L \tag{1}$$

$$u(0,t) = u_{xx}(0,t) = 0 \quad u(L,t) = u_{xx}(L,t) = 0, \quad t \geq 0 \tag{2}$$

$$u(x,0) = f(x), \quad u_t(x,0) = 0 \quad 0 \leq x \leq L \tag{3}$$

a. Find the formal solution of the above problem

b. Find restrictions on the function $f(x)$ so that the formal solution in part **a** is a solution of the problem.

c. Prove the uniqueness of the solutions of the above problem. (Hint: solve this question without considering the first two parts **a** and **b** and use the energy integral .

$$E(t) = \frac{1}{2} \int_0^L [(u_t)^2 + a^2 (u_{xx})^2] dx$$