MATH 544: METHODS OF APPLIED MATHEMATICS II
First Midterm Exam
March 14, 2016; 10.40-12.30

QUESTIONS: Choose any three of the following four problems

[35] **1**. **(a)** A functional is given as

$$J[y] = \int_0^{\ln 2} [y^2 + (y')^2] dx$$

with the boundary conditions $y(0) = 0$ and $y(\ln 2) = 3/4$. Find the function y which is the critical point of this functional. **(b)** Determine whether the critical point you found in part (a) minimizes or maximizes the functional $J[y]$.

[35] **2**. An initial value problem is given as follows: Is there a unique solution? If the answer is negative find conditions on data to have solutions.

$$\begin{aligned} &u_{tx} + u_t = 1, \quad t > 0, x \in \mathbb{R} \\ &u(x,0) = f(x), \quad x \in \mathbb{R} \\ &u_t(x,0) = g(x), \quad x \in \mathbb{R} \\ &u_x(x,0) = h(x), \quad x \in \mathbb{R} \end{aligned}$$

where f, g and h are given functions.

[35] **3**. An initial and boundary value problem is given as follows:

$$\begin{aligned} &u_{tt} - c^2\, u_{xx} = 0, \quad t > 0, \quad 0 < x < L \\ &u(x,0) = f(x), \quad u_t(x,0) = g(x), \quad 0 \le x \le L \\ &u_x(0,t) = u_x(L,t) = 0, \quad t \ge 0, \end{aligned}$$

where f and g are given functions. Under what conditions this problem is a well posed problem?

[35] **4**. If l is not preassigned, show that the stationary functions corresponding to the problem $\delta \int_0^l [y'^2 + 4(y - l)] dx = 0$, with $y(0) = 2$ and $y(l) = l^2$ are of the form $y = x^2 - 2(x/l) + 2$, where l is one of two real roots of the equation $2l^4 - 2l^3 - 1 = 0$. (Hint: This is a moving end point problem)