MATH 544: METHODS OF APPLIED MATHEMATICS II

Second Midterm Exam

April 28, 2014; 13.40-15.30

QUESTIONS:

[50] **1**. (a). Solve

$$y(x) = f(x) + \lambda \int_0^\infty [\alpha\, e^{-x-t} + \beta\, e^{-2x-2t}]\, y(t)\, dt$$

where α and β are constants. Discuss all possible cases. (b) Find all solutions when $f(x) = 1 - x$.

[50] **2**. (a). Solve following Dirichlet problem

$$\nabla^2 u = 0, \quad \text{in } D,$$
$$u|_{boundary} = f(x,y), \quad (x,y) \in \mathbb{R}^2$$

where D is the region $z > 0$ in $\mathbb{R}^3$ with boundary $z = 0$. (b). Generalize this Dirichlet problem to $\mathbb{R}^n$.

$$\nabla^2 u = 0, \quad \text{in } D \in \mathbb{R}^n,$$
$$u|_{boundary} = f(x), \quad x \in \mathbb{R}^{n-1}$$

where D is the region $z > 0$ in $\mathbb{R}^n$ with boundary $z = 0$.