MATH 544: METHODS OF APPLIED MATHEMATICS II
Second Midterm Exam
April 28, 2014; 13.40-15.30

QUESTIONS:

[50] 1. (a). Solve
\[ y(x) = f(x) + \lambda \int_0^\infty \left[ \alpha e^{-x-t} + \beta e^{-2x-2t} \right] y(t) \, dt \]
where \( \alpha \) and \( \beta \) are constants. Discuss all possible cases. (b) Find all solutions when \( f(x) = 1 - x \).

[50] 2. (a). Solve following Dirichlet problem
\[ \nabla^2 u = 0, \quad \text{in } D, \]
\[ u|_{\text{boundary}} = f(x, y), \quad (x, y) \in \mathbb{R}^2 \]
where \( D \) is the region \( z > 0 \) in \( \mathbb{R}^3 \) with boundary \( z = 0 \). (b). Generalize this Dirichlet problem to \( \mathbb{R}^n \).

\[ \nabla^2 u = 0, \quad \text{in } D \in \mathbb{R}^n, \]
\[ u|_{\text{boundary}} = f(x), \quad x \in \mathbb{R}^{n-1} \]
where \( D \) is the region \( z > 0 \) in \( \mathbb{R}^n \) with boundary \( z = 0 \).