MATH 544: METHODS OF APPLIED MATHEMATICS II

Final Exam

May 21, 2013; 9:00-11:00 (SA-Z03)

QUESTIONS:

[35]**1.** Solve the following initial value problems: **(a)**: $u_{xx} - 3u_{xy} + 2u_{yy} = 0$ and $u(x,x) = f(x)$, $u_y(x,x) = g(x)$ **(b)**: $u_{xx} - 3u_{xy} + 2u_{yy} = 0$ and $u(x,-x) = f(x)$, $u_y(x,-x) = g(x)$

[35]**2.** A boundary value problem is given by

$$\nabla^2 u = 0, \quad z > 0, \ (x,y) \in \mathbb{R}^2,$$
$$u(x,y,0) = f(x,y), \quad (x,y) \in \mathbb{R}^2,$$

and assume that u is finite everywhere $z \geq 0$

(a)) Solve this Dirichlet's problem by assuming $f(x,y) = f(r)$ where $r^2 = x^2 + y^2$. (Hint: Use cylindrical coordinates to solve this problem. The Bessel function $J_\nu(x)$ satisfies the Bessel equation $y'' + \frac{1}{x}\,y' + (1 - \frac{\nu^2}{x^2})y = 0$)

(b) Solve the above boundary value problem by using the method of Green's function. In this case $f = f(x,y)$ not $f \neq f(r)$.

[35]**3.** **(a)** Obtain the most general solution of the integral equation

$$y(x) = f(x) + \lambda \int_{-1}^{1} g(x,s)\, y(s) ds$$

with

$$g(x,s) = \sum_{k=0}^{\infty} a_k\, P_k(x)\, P_k(s)$$

for any f. Here a_k's are constants and $P_n(x)$ $(n = 0, 1, \cdots)$ are the Legendre polynomials

(b) Solutions when

$$f(x) = \sum_{n=0}^{\infty} b_n\, P_n(x).$$

Discuss all possible cases.

[You may need the formula:

$$\int_{-1}^{1} P_n(x)\, P_m(x)\, dx = \frac{2}{2n+1}\, \delta_{mn}]$$