MATH 544: METHODS OF APPLIED MATHEMATICS II

First Midterm Exam

March 19, 2013; 8.40-10.30

QUESTIONS:

[30] **1**. An initial value problem is given as

$$\begin{aligned} &u_{tt} - u_{xx} = 0, \\ &u(x,x) = f(x), \quad u_t(x,x) = g(x) \end{aligned}$$

where $f(x)$ and $g(x)$ are given function. Find condition(s) on $f(x)$ and $g(x)$ so that the problem has solutions.

[30] **2**. An initial and boundary value problem is given as

$$\begin{aligned} &u_{tt} - c^2\, u_{xx} = 0, \quad -L < x < L, \quad t > 0, \\ &u_x(0,t) = u_x(L,t) = 0, \quad t \geq 0, \\ &u(x,0) = f(x), \quad u_t(x,0) = 0, \quad -L \leq x \leq L \end{aligned}$$

where $f(x)$ is a given function. Find conditions on $f(x)$ so that the problem is well defined.

[30] **3**. Find the curve(s) for which the functional

$$J[y] = \int_0^a \frac{\sqrt{1+y'^{\,2}}}{y}\, dx, \quad y(0) = 0, \tag{1}$$

have extrema if the point (a,b) vary along the line $y = x - 5$

[30] **4**. Find the extrema of the problem

$$\delta \int_0^\pi (y'')^2\, dx = 0, \quad y(0) = y''(0) = 0, \quad y(\pi) = y'(\pi) = 0 \tag{2}$$

with

$$\int_0^\pi y^2 dx = 1$$