MATH 544: METHODS OF APPLIED MATHEMATICS II

Final Exam

May 21, 2012; 15.30-17.30 (SAZ-02)

QUESTIONS:

[25] **1**. Let

$$\begin{aligned} \frac{dx}{dt} &= 2x - 3y + 1, \\ \frac{dy}{dt} &= x - 2y - 1 \end{aligned}$$

(a) Find the equilibrium solutions of the above system.
(b) Plot the phase portrait and determine the stability types of the equilibrium points.

[25] **2**. Let

$$\frac{du}{dt} = (u-2)(u^2 - \mu),$$

where μ is a real constant. (a) Determine the bifurcation points and classify them. (b) Determine the stability types of the equilibrium points (c) Plot the bifurcation diagram.

[25] **3**. Prove the mean value theorem, in $\mathbb{R}^2$, for the Laplace equation. By the use of the mean value theorem prove also the maximum-minimum theorem of the Laplace equation.

[25] **4**. Let a functional J be defined by

$$J = \int_a^b L(x, y(x), y_x(x))dx$$

where L is the Lagrange function, y is a function of x, and $y_x = \frac{dy}{dx}$. Prove that any two Lagrange functions differ by a total derivative are equivalent, i.e., Euler-Lagrange equations give the same ODE.