QUESTIONS:

[25] 1. Let
\[ \frac{dx}{dt} = 2x - 3y + 1, \]
\[ \frac{dy}{dt} = x - 2y - 1 \]
(a) Find the equilibrium solutions of the above system.
(b) Plot the phase portrait and determine the stability types of the equilibrium points.

[25] 2. Let
\[ \frac{du}{dt} = (u - 2)(u^2 - \mu), \]
where \( \mu \) is a real constant. (a) Determine the bifurcation points and classify them. (b) Determine the stability types of the equilibrium points (c) Plot the bifurcation diagram.

[25] 3. Prove the mean value theorem, in \( \mathbb{R}^2 \), for the Laplace equation. By the use of the mean value theorem prove also the maximum-minimum theorem of the Laplace equation.

[25] 4. Let a functional \( J \) be defined by
\[ J = \int_a^b L(x, y(x), y_x(x)) \, dx \]
where \( L \) is the Lagrange function, \( y \) is a function of \( x \), and \( y_x = \frac{dy}{dx} \). Prove that any two Lagrange functions differ by a total derivative are equivalent, i.e., Euler-Lagrange equations give the same ODE.