MATH 544: METHODS OF APPLIED MATHEMATICS II

Second Midterm Exam

April 30, 2012; 13.40-15.30 (SAZ-02)

QUESTIONS:

[40] **1**. Consider the integral equation

$$y(x) = f(x) + \lambda \int_0^1 (1 - k\,x\,s) y(s) ds$$

where $f(x)$ is a continuous function in $[0,1]$, k is a constant and λ is the parameter of the integral equation. (i) Find the solution (ii) find the cases (restrictions on λ where there is no solution and (iii) the solution is not unique (iv) find the solution when $f(x) = x^2$. Follow the cases (i)-(iv) and answer them. Discuss all possible cases.

[40] **2**. Let $u \in C^2(D) \cap C^1(\bar{D})$, where D is a bounded domain and B is the boundary of D.

(i) If $\nabla^2 u \geq 0$ in D, then $u(x) \leq M = max_{y \in B}\, u(y)$ for all $x \in D$.
(ii) If $\nabla^2 u \leq 0$ in D, then $u(x) \geq m = \min_{y \in B}\, u(y)$ for all $x \in D$.
(iii) If$\nabla^2 u = 0$ in D, then $m \leq u(x) \leq M$ for all $x \in D$.

[40] **3**. Let $u(x,t)$ be a solution to the problem

$$u_t - a u_{xx} = 0, \quad t > 0,\ 0 \leq x \leq L, \tag{1}$$

$$u(0,t) = u(L,t) = 0, \quad t \geq 0 \tag{2}$$

$$u(x,0) = f(x), \quad 0 \leq x \leq L \tag{3}$$

let $\tilde{u}$ be a solution of the above problem with f replaced by $\tilde{f}$. Denote the respective Fourier coefficients by α_n and $\tilde{\alpha_n}$. Show $|\alpha_n - \tilde{\alpha_n}| \leq 2||f'' - \tilde{f}''||/\lambda_n^2$ and hence

$$||u - \tilde{u}|| \leq 2||f'' - \tilde{f}''|| \left(\sum_{n=1}^{\infty} \frac{1}{\lambda_n^2} \right)$$

where $||h|| = max|h(x)|$ for $0 \leq x \leq L$. Deduce continuous dependence and hence the well-posedness of the problem.