MATH 544: METHODS OF APPLIED MATHEMATICS II

First Midterm Exam

March 19, 2012; 13.40-15.30

QUESTIONS:

[40] **1**. An initial and boundary value problem is given as

$$\begin{aligned} &u_{tt} - c^2\, u_{xx} = 0, \quad -L < x < L, \quad t > 0, \\ &u(0,t) = u_x(L,t) = 0, \quad t \geq 0, \\ &u(x,0) = f(x), \quad u_t(x,0) = 0, \quad -L \leq x \leq L \end{aligned}$$

where $f(x)$ is a given function. Find conditions on $f(x)$ so that the problem is well defined.

[40] **2**. Solve the following variational problem

$$\delta \left[\int_0^b [\lambda^2\, \dot{y}^2 - y^2] dt - 2w^2 y(b) \right] \tag{1}$$

with $y(0) = 0$. Here λ and w are arbitrary constants. Calculate the functional at the solution and determine also whether this value corresponds to a local maximum or minimum value for the functional.

[30] **3**. Solve the initial value problem $u_{xy} = F(x,y)$ with $u(x,x) = f(x), u_y(x,x) = g(x)$, where $F(x,y)$, $f(x)$ and $g(x)$ are given functions (data). (Note: you can solve this problem in many ways. If you solve it by using the Riemann method I will give 40 points.)