HOMEWORK VI (MATH 544): *April 18, 2012*
(For May 2, 2012)

1. 8. In two dimensions consider

$$\nabla^2 u = 0, \quad 0 < x < a, \quad 0 < y < b, \tag{1}$$
$$u(x,0) = f(x), \quad u(x,b) = 0, \quad 0 \le x \le a, \tag{2}$$
$$u(0,y) = u(a,y) = 0, \quad 0 \le y \le b. \tag{3}$$

a. Find the formal solution,
b. Find reasonable restrictions on $f(x)$ so that the formal solution is a solution.

2. Find Green's function of the following problem

$$-k\,u'' + l\,u = 0, \quad u(0) - u'(0) = 0, \quad u(1) = 0.$$

Find the solution of the boundary value problem.

3. Show that the Green's function is unique, if it exists. *Hint.* If both $G(x,y)$ and $H(x,y)$ provides the same $u(x)$ given by

$$u(x) = \int_a^b G(x,y) f(y) dy, \tag{4}$$
$$u(x) = \int_a^b H(x,y) f(y) dy, \tag{5}$$

then

$$\int_a^b [G(x,y) - H(x,y)] f(y) dy = 0$$