HOMEWORK V (MATH 544): *April, 2012*
(For April 9, 2012)

1. For $T > 0$ let

$$Q_T = \{(x,t)|0 < x < L,\ 0 < t \leq T\}$$

(this is exactly the region D defined in the lecture), be a rectangle with its bottom and two vertical sides removed. Let

$$\bar{Q}_T = \{(x,t)|0 \leq x \leq L,\ 0 \leq t \leq T\}.$$

Then $B_T = \bar{Q}_T - Q_T$ consists of the three missing sides of the original rectangle Q_T. Suppose that $w(x,t)$ satisfies the differential inequality

$$w_t - a(x,t)w_{xx} - b(x,t)w_x - c(x,t)w < 0 \quad (x,t) \in Q_T$$

where $a(x,t) \geq 0$ and $c(x,t) \leq 0$ in Q_T. Then $w(x,t)$ can not achieve a positive local maximum in Q_T

2. Let a, b and c be as in the previous problem. Suppose that $u(x,t)$ is continuous on $\bar{Q}_T$ and satisfies

$$u_t - a(x,t)u_{xx} - b(x,t)u_x - c(x,t)u \leq 0, \quad (x,t) \in Q_T, \tag{1}$$

$$u(x,t) \leq 0, \quad (x,t) \in B_T \tag{2}$$

Then $u(x,t) \leq 0$ on $\bar{Q}_T$.

3. Let a, c and c be as in Prob.4. Suppose that $u(x,t)$ is continuous on $\bar{Q}_T$ and satisfies

$$u_t - a(x,t)u_{xx} - b(x,t)u_x - c(x,t)u = 0$$

in Q_T. If u has a positive maximum , it is assumed on the boundary B_T and if u has a negative minimum , it is assumed on B_T.

4. Let a, b and c be as in Prob. 4. Suppose that $F(x,t)$ is defined in Q_T and bounded by the constant N there. Suppose that $u(x,t)$ is continuous on $\bar{Q}_T$, $M = max_{B_T}|u(x,t)|$, and $u(x,t)$ satisfies

$$u_t - a(x,t)u_{xx} - b(x,t)u_x - c(x,t)u = F(x,t)$$

in Q_T. The $|u(x,t)| \leq M + TN$ in Q_T.

5. Let $a(x,t) > 0$ and $c(x,t) \leq 0$ for all $0 < x < L$ and $t >$). Show that the initial boundary value problem

$$u_t - a(x,t)u_{xx} - b(x,t)u_x - c(x,t)u = F(x,t), \quad 0 < x, L, \quad t > 0, \tag{3}$$

$$u(0,t) = g(t), \quad u(L,t) = h(t), \quad t \leq 0, \tag{4}$$

$$u(x,0) = f(x), \quad 0 \leq x \leq L \tag{5}$$

has at most one solution which is continuous on $0 \leq x \leq L$ and $t \geq 0$ [Notice that if there is , in fact a continuous solution, then the boundary and initial data must be compatible. That is ,$g(0) = f(0), \quad h(0) = f(L)$].