HOMEWORK III (MATH 544): *March 7, 2012*
(For March 21, 2012)

1. An initial value problem of the wave equation:

$$\begin{aligned} &u_{tt} - c^2\, u_{xx} = 0, \quad t > 0,\ 0 < x < L, \\ &u(0,t) = 0, \quad u_x(L,t) = 0, \quad t \geq 0, \\ &u(x,0) = f(x), \quad u_t(x,0) = g(x), \quad 0 \leq x \leq L \end{aligned}$$

where $f(x)$ and $g(x)$ are given functions (data) for all $x \in (0, L)$. Find the conditions on the data so that the above initial value problem is well posed.

2. Solve the following initial value problems.

$$x^2\, u_{xx} + 3xy\, u_{xy} + 2y^2\, u_{yy} = 0, \quad (x,y) \in D \subset \mathbb{R}^2 \tag{1}$$

with the following initial data:
(a) $u(x, x\sqrt{x}) = f(x)$ and $u_n(x, x\sqrt{x}) = g(x)$ where $f(x)$ and $g(x)$ are given functions of x and u_n is the directional derivative of u along the normal vector of the initial curve.

(b) $u(x,x) = f(x)$ and $u_n(x,x) = g(x)$ where $f(x)$ and $g(x)$ are given functions of x and u_n is the directional derivative of u along the normal vector of the initial curve.