HOMEWORK II (MATH 544): *February 22, 2012*
(For March 5, 2012)

1. Obtain the Euler-Lagrange equation and the associated natural boundary conditions for the problem $\delta J = 0$ where

$$J(y) = \int_a^b L(x, y, y')dx - \beta y(b) + \alpha y(a)$$

Here α and β are arbitrary constants and $y(a)$ and $y(b)$ are not prescribed.

2. Find the shortest distance between the line $y = x$ and the parabola $y^2 = x - 1$.

3. If b is not prescribed (moving end point), show that the stationary functions corresponding to the problem $\delta J = 0$ where

$$J(y) = \int_0^b \left[(y')^2 + 4(y - b)\right] dx$$

with $y(0) = 2$ and $y(b) = b^2$ are of the form $y = x^2 - 2(x/b) + 2$, where b is one of the real roots of the equation $2b^4 - 2b^3 - 1 = 0$.

4. Obtain the stationary functions of the problem $\delta J = 0$ where

$$J(y) = \int_0^\pi \left[\lambda^2 (y')^2 - w^2\, y^2\right] dx$$

with $y(0) = 0$, where λ and w are constants. Determine whether J has minimum or maximum value for these stationary functions.