HOMEWORK I (MATH 544): *February 13, 2012*
(For February 20, 2012)

1. Find the Euler-Lagrange equation and also the natural boundary conditions for the functional $J(y) = \int_a^b L(x, y, y', y'')\, dx$ with the boundary conditions

$$y(a) = A_1, \quad y'(a) = A_2, \tag{1}$$
$$y(b) = B_1, \quad y'(b) = B_2 \tag{2}$$

where we assume $y \in C^4(a, b)$ and Lagrange function L is a twice differentiable function in $(a, b) \times \mathbb{R}^3$. Here $y' = \frac{dy}{dx}$ and A_1, A_2, B_1, B_2 are given real numbers.

2. Show that when the Lagrange Function L is a total derivative, i.e., $L = \frac{d\Omega}{dx}$ the Euler-Lagrange equation is identically satisfied. Assume that $\Omega = \Omega(x, y, y')$ is a twice differentiable function of its arguments.