MATH 543

METHODS OF APPLIED MATHEMATICS I
Second Midterm Exam

April 24, 2017
Monday 10.40-12.30

PROBLEMS: Choose any three of the following four questions:
[35]**1**. A boundary value problem is given by

$$\mathrm{L}(u) = f(x), \quad x \in [a,b], \tag{1}$$
$$B_1(u) = 0, \quad B_2(u) = 0 \tag{2}$$

where ord(L)=2 and L, B_1 and B_2 are linear operators and L is hermitian. Prove that the solution of this boundary value problem exists provided that the solutions of the homogenous equation has no nontrivial solution satisfying both boundary conditions. Here f is a continuous function in $[a,b]$.

[35]**2**. By using the method of Greens function solve the boundary value problem $u'' = f(x)$ where $x \in [0,1]$ with $u(0) = \alpha$ and $u'(1) = \beta$ (Here α and β are real constants). Here $f(x)$ is any continuous function in $[0,1]$.

[35] **3**. Let $J(y) = \int_a^b L(x,y,y')dx$ where $y(a) = y_0$ and $y(b) = y_1$. Here L is a twice differentiable function of its arguments and $y \in C^2[a,b]$. Find the necessary condition for J to have an extremum for a given function y.

[35]**4**. Let $J(y) = \int_0^1 [(y')^2 + k^2y^2]dx$ with $y(0) = 0$ and $y(1) = 1$. Here k is nonzero real constant. Show that $y_0 = \frac{\sinh(kx)}{\sinh(k)}$ minimizes the functional J.