MATH 543
METHODS OF APPLIED MATHEMATICS I
Second Midterm Exam

December 4, 2015
Friday 10:40-12:30

PROBLEMS: Choose any three of the following five questions:

1[35].(a) Show that every linear second order differential operator $L = a(x)\frac{d^2}{dx^2} + b(x)\frac{d}{dx} + c(x)$ with real coefficients is self adjoint provided that the weight function $w(x)$ is chosen as $w(x) = \frac{1}{a(x)}\, exp\left(\int^x dx' \frac{b(x')}{a(x')}\right)$ and $a(x) > 0$.
(b) Show also that the self adjoint operator L in a boundary value problem $L(u) = f(x),\ \ x \in (a,b)$ with the boundary conditions $\alpha u(a) - u'(a) = 0$ and $\beta u(b) - u'(b) = 0$ is Hermitian(Here α and β are real constants).

2[35]. By using the method of Green's function find the solution of the following boundary value problem:

$$u'' + u' = f(x), \quad x \in (0,1),$$
$$u(0) = 1, \quad u'(0) = -1$$

3[35]. Prove the existence and the uniqueness of the Green's function of boundary value problem

$$L(u) = f(x), \quad x \in (a,b),$$
$$B_1(u) = 0, \quad B_2(u) = 0$$

where L is a linear second order hermitian operator (with real coefficients) and the corresponding homogenous equation has no nontrivial solutions satisfying both boundary conditions (Here B_1 and B_2 are also linear operators).

4[35]. Find the power series solution of the differential equation $x(1-x)u'' - 2u' + 2u = 0$ about $x = 0$.

5[35]. Let $z = z_0$ be regular singular point of the differential equation $u'' + p(z)\,u' + q(z)\,u = 0$ with the indices r_1 and r_2. If $r_1 - r_2 = N$ a positive integer then show that the solution corresponding to $r = r_2$ takes the form

$$u_2(z) = C\,u_1(z)\,\ln(z - z_0) + (z - z_0)^{r_2} \sum_{n=0}^{\infty} C_n\,(z - z_0)^n$$

where C is a constant and $u_1(z)$ is the solution corresponding to $r = r_1$.