MATH 543

METHODS OF APPLIED MATHEMATICS I
First Midterm Exam

October 30 2015
Friday 10:40-12:30

PROBLEMS: Choose any three of the following four questions:

1[35]. Assume that there exists an orthonormal basis $|e_i>$, $(i = 1, 2, \cdots)$ in $L^2_w(a, b)$. Then show that , for any $|f> \in L^2_w(a, b)$, the sequence of vectors

$$|f_k> = \sum_{i=1}^{k} f^i \, |e_i>, \quad (k = 1, 2, \cdots)$$

with

$$f^i = < e_i | f >, \quad (i = 1, 2, \cdots)$$

has $|f>$ as the limit vector in the sense that

$$\lim_{k\to\infty} \rho(|f>, |f_k>) = 0$$

where ρ is the distance function of $L^2_w(a, b)$,

2[35]. Let $|f>$ be a vector in $L^2_e(a, b)$. Prove that the set of orthonormal vectors $\{|e_i>\}, (i = 1, 2, \cdots)$ form a basis of $L^2_e(a, b)$ if and only if the Fourier coefficients of $|f>$ with respect to $|e_i>$ satisfy the Parseval's relation.

3[35]. Prove that the classical orthogonal polynomial $C_n(x)$ with weight function $w(x)$ and $x \in [a, b]$ has n distinct real roots in $[a, b]$.

4[35]. Let $\delta(x)$ be the Dirac δ-function. Prove that

$$\int_{-\epsilon}^{\epsilon} \delta(x - a) \, f(x) \, dx = f(a), \quad (\epsilon > 0)$$

whenever $-\epsilon < a < \epsilon$