MATH 543
METHODS OF APPLIED MATHEMATICS I
First Midterm Exam

November 01, 2011
Thuesday 13:40-15:30, SAZ-02

QUESTIONS:
[35]**1**.
1.1. Prove that Fourier transform of a good function is also a good function.
1.2. If $\delta(x)$ represents the Dirac delta function prove that

$$\int_{-\infty}^{\infty} x\delta(x-a)dx = a$$

where a is any real number.
1.3. Prove that

$$\delta(\lambda x) = \frac{1}{|\lambda|}\,\delta(x)$$

for any real number $\lambda \neq 0$.

[35]**2**. Prove that the set of orthonormal vectors $|e_i>$ $(i = 1, 2, ...)$ form a basis of $L^2_w(a, b)$ if and only if the Fourier coefficients with respect to the basis $|e_i>$ satisfy Parseval's relation

[35]**3**. Let

$$C_n(x) = \frac{1}{w(x)}\,\frac{d^n}{dx^n}\,(w(x)s(x)^n), \quad n = 0, 1, 2, \cdots, x \in [a, b]$$

where
(a) $w(x) > 0$ for all $x \in [a, b]$.
(b) $s(x)$ is a polynomial of degree ≤ 2 with real distinct roots.
(c) $C_1(x)$ is a polynomial of degree 1.
(d) $w(a)s(a) = w(b)s(b) = 0$ (boundary conditions).
3.1. Prove that C_n's are polynomials of degree n.
3.2. Prove that C_n's are orthogonal to any polynomial of degree less then n.(Orthogonality with respect to the $L^2_w(a, b)$ inner product).
3.3. Show that C_n's satisfy a second order linear differential equation.