MATH 543

METHODS OF APPLIED MATHEMATICS I
Second Midterm Exam

December 12, 2013
Thursday 10.30-12.20

PROBLEMS:

1[40]. *The method of Green's function*: Let $u'' = f(x), \quad x \in (0,1)$ with $u(0) - \alpha\, u'(0) = \gamma_1$ and $u(1) - \beta\, u'(1) = \gamma_2$. Here α, β, γ_1 and γ_2 are given constants and $f(x)$ is a given function. Solve this boundary value problem by the use of Green's function method.

2[40]. *Solutions in the neighborhood of a regular singular point*: Determine the two values of the constant α for which it is true that all solutions of the equation $z\, u'' + (z-1)\, u' - \alpha\, u = 0$ are regular at $z = 0$, and obtain the general solution in each of these cases.

3[40].*Existence and uniqueness of solutions in the neighborhood of an ordinary point*: Let $u'' + p(z)\, u' + q(z) u = 0$ where $p(z)$ and $q(z)$ are analytic functions in a neighborhood of a point $z = z_0$ (an ordinary point of the DE) with the initial conditions $u(z_0) = u_0$ and $u'(z_0) = u_1$. Here u_0 and u_1 are given numbers. Prove that the solution of the differential equation exists and unique.