MATH 543: METHODS OF APPLIED MATHEMATICS I

Makeup Exam:

07 January 2002 Monday 20.00-20.00SAZ:18

QUESTIONS:

(25)**1**. Find a uniform approximate solution of the following boundary value problem :

$$\varepsilon u'' + (1+\varepsilon)\, u' + u = 0, \quad 0 < t < 1, \quad 0 < \varepsilon << 1$$

with $u(0) = 0, \quad u(1) = 1$.

(25)**2**. Use the method of Frobenius to obtain the general solution of the following differential equation, valid near $z = 0$.

$$z\, u'' - u' + 4z^3\, u = 0$$

(25)**3**. Prove that all second order Fuchsian linear differential equations having only two singular points are equivalent to second order linear differential equations with constant coefficients. Hence they are solvable in terms of either exponential and trigonometric functions.

(25)**4**. Let $L_x\, u = A(x)\, u'' + B(x)\, u' + C(x)\, u(x) = 0$ where $A(x), C(x)$ are continuous and $B(x)$ is differentiable in $[a, b]$. Let u and v be in $L^2_w(a, b)$ and the adjoint operator $L^\dagger_x$ of L_x be defined by

$$w[\bar{v} L_x u - u\overline{\{L^\dagger_x v\}}] = \frac{d}{dx}\, Q(u, \bar{v})$$

where $w > 0$ is the weight function and Q is the surface term.

(a) Prove that everey liner second order differential operator L_x with real coefficients is self-adjoint provided the weight function $w(x)$ is choosen properly.

(b) Prove that the operator L_x defined in part (a) with the boundary conditions $u'(a) = u'(b) = 0$ on $u(x)$ defines a Hermitian differential operator L.