MATH443
PARTIAL DIFFERENTIAL EQUATIONS
MAKEUP EXAM

January 10, 2011 Friday 09.00-11.00, SAZ18

QUESTIONS: Solve any four of the following five problems. *In each problem explain what you are doing.*

[25]**1**. Find the solution of the following initial and boundary value problem.

$$\begin{aligned} u_t = k u_{xx}, \quad 0 < x < 1, \quad t > 0, \\ u(0,t) = 0, \quad u_x(1,t) = 0, \quad t \geq 0 \\ u(x,0) = \sin \frac{\pi\, x}{2} - 7 \sin \frac{3\pi x}{2} \end{aligned}$$

[25]**2**. Find a complete integral of the equation $p^2\, x + q^2\, y = z$.

[25]**3**. Reduce the equation

$$y^2\, z_{xx} - 2xy\, z_{xy} + x^2\, z_{yy} = \frac{y^2}{x}\, z_x + \frac{x^2}{y}\, z_y$$

to canonical form, and hence solve it.

[25]**4**. Let ψ satisfy a boundary value problem of the Laplace equation in a circular disk D of radius R in the plane. Prove that ψ can not take its extremum values in D.

[25]**5**. Solve the following equation

$$z_{xxx} - 3\, z_{xxy} + 2\, z_{xyy} = 6\, e^{3x+y}, \tag{1}$$

with the conditions $z(0,y) = e^y$, $z_x(0,y) = 0$ and $z_{xx}(0,y) = 0$.