MATH 346
DIFFERENTIAL GEOMETRY II
Second Midterm Exam

April 22, 2003
Tuesday 08.40-10.30, SAZ-19

QUESTIONS: Solve any four of the following five questions

[25]**1.** Let $\alpha : I \to M$ be a curve in $M \in \mathbb{R}^3$. Let Y be a vector field on α tangent to M. Prove that

$$\dot{Y} = Y' + [S(\alpha') \cdot Y]\, U$$

where $\dot{Y}$ denotes the Euclidean derivative of Y. Hence the Euclidean acceleration of α is

$$\ddot{\alpha} = \alpha'' + [S(\alpha') \cdot \alpha']\, U$$

[25]**2.** Using the geographical patch

$$X(u, v) = (r \cos v \, \cos u, r \cos v \, \sin u, r \sin v)$$

where $0 < u < 2\pi,\ 0 < v < \pi$ of a sphere of radius r, **(a)** find a frame $\{E_1, E_2\}$, **(b)** find its dual frame $\{\theta_1, \theta_2\}$, **(c)** find the connection 1-form **(d)** Find the Gauss curvature K and **(e)** calculate the total curvature of the sphere.

[25]**3.** Prove that on every compact surface in $\mathbb{R}^3$ there exist a point at which the Gaussian curvature K is strictly positive.

[25]**4.** Prove that a simple closed curve α in $\mathbb{R}^3$ has a total curvature

$$\int_\alpha \kappa \, ds \ \geq \ 2\pi$$

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[25]**5.** A mapping of surfaces $F : M \to N$ is conformal provided there exists a real valued function $\lambda > 0$ on M such that

$$||F_\star (v_p)|| = \lambda(p)||v(p)||.$$

Prove that a regular mapping $F = (f, g)$ from a region $U \subset \mathbb{R}^2$ to $\mathbb{R}^2$ is conformal and orientation preserving if and only if the Cauchy-Riemann equations $f_u = g_v,\ f_v = -g_u$ hold.