MATH 346
DIFFERENTIAL GEOMETRY II
First Midterm Exam

March 17, 2008
Monday 10.40-12.30, SAZ-19

QUESTIONS: Solve any three of the following four questions

[35]**1**. (**a**). Let S be an oriented surface of negative or zero curvature. Prove that two geodesics which start from a point $p \in S$ cannot meet again at another point $q \in S$ in such a way that the the geodesic curves constitute the boundary of a simple region of S. (**b**). On this surface S prove that a smoothly closed geodesic can not bound a simple region. (**c**). Consider a triangular simple region T in S. Apply the Gauss-Bonnet theorem to the region T and find the total interior angles of T. Discuss your result.

[35]**2. a)**. Let S be the right cylinder parametrized as $X(u, v) = (\cos u, \sin u, v)$, $(0 \leq u < 2\pi, v \in \mathbb{R})$. Find the first and second fundamental forms, Christoffel symbols, Gaussian and mean curvatures of the Cylinder. **b)**. Write the geodesic equations of the cylinder and find all geodesics of cylinder by the use of these geodesic equations. (**c**). Show that the coordinate curves of the cylinder are geodesics of the cylinder.

(35)**(3).** Let p_0 be a pole of unit sphere S^2
and q, r be two points on the corresponding
equator in such a way that the meridians p_0q
and p_0r make an angle θ at p_0. Consider
a unit vector v tangent to meridian p_0q at p_0 ,
and take the parallel transport of v along the closed curve made up by the meridian p_0q, the parallel qr , and the meridian rp_0. See the figure.
(a). Determine the angle of the final position of v with v. **(b).** Do the same thing when the points r, q instead of being on the equator are taken on a parallel of colatidute ϕ. **(c)**. Let C be the parallel of colatidute ϕ. Apply the Gauss-Bonnet theorem to find the area of the (upper) region bounded be C
.

[35]**4.** Let $X : U \to S$ be an orthogonally parametrized regular surface. If the coordinate curves $(X(u, v_0), X(u_0, v),$ where $(u, v), (u_0, v_0) \in U)$ are the geodesics of S prove that the Gaussian curvature of S is zero everywhere.