MATH 346
DIFFERENTIAL GEOMETRY II
First Midterm Exam

March 11 , 2003
Tuesday 08.40-10.30, SAZ-19

Surfaces of Revolution. Let $\alpha(t) = (g(t), h(t), 0)$ be a plane curve in $\mathbf{R}^3$ (*profile curve*) where $t \in I$. Rotating this curve about the $x-$ axis we obtain a surface M (surface of revolution) with the parametrization

$$X(t, v) = (g(t), h(t)\cos v, h(t)\sin v)$$

with $h(t) > 0$, where $(t, v) \in D = \{t \in I, 0 < v < 2\pi\}$. The coordinate curves are the meridians defined by $\beta(t) = X(t, v_0)$ and are the parallels defined by $\gamma(v) = X(t_0, v)$ where $t_0 \in I$ and $v_0 \in (0, 2\pi)$ are some fixed real numbers.

QUESTIONS

1. Find the coefficients of the first and the second fundamental forms of M.

2. Prove that both of the coordinate curves are principal curves of M.

3. Under what conditions the coordinate curves are asymptotic curves of M.

4. Under what conditions the coordinate curves are geodesics of M.

5. Find the principal , the Gaussian and the mean curvatures of M.

6. Find all surfaces of revolutions with zero Gaussian curvature.