MATH 345
DIFFERENTIAL GEOMETRY I
Second Midterm Exam

December , 2002
Friday 18.00-20.00, SAZ-18

QUESTIONS:

[25]**1**. (a) Let $F : M \to N$ be a regular mapping. Prove that if N is orientable then M is orientable. Explain why F must be a regular mapping. (b) Let $f : M \to \mathbf{R}$ be a nowhere vanishing continuous function on M Prove that F is either positive or negative on M

[25]**2**. Let $X : R \to M$ be a 2$-$ segment defined on a unit square $R = \{(u, v) \in \mathbf{R}^2 | 0 \leq u \leq 1, 0 \leq v \leq 1\}$. If ϕ is 1$-$ form on M such that $\phi(X_{,u}) = u+v$ and $\phi(X_{,v}) = uv$. (**a**) Compute $X^* \, \phi$ and $X^* \, d\phi$ (**b**) Compute $\int \int_X \, d\phi$ and $\int_{\partial X} \phi$ separetely and check the result by Stoke's theorem.

[25]**3**. Let $M = \{(x, y, z) \in \mathbf{R}^3 | f(x, y, z) = c\}$ be a regular surface in $\mathbf{R}^3$, where c is a real constant. (**a**) What are the conditions on the function f?. (**b**) Prove that the gradient vector field, ∇f, (considered only on M) is a (nonvanishing) normal vector field on M. (**c**) Prove that M is orientable.

[25]**4**. Prove that the sphere S^2 is a two-dimensional smooth manifold.