MATH 345
DIFFERENTIAL GEOMETRY
Final Exam

November 8 , 2002
Friday 18.00-20.00, SAZ-04

QUESTIONS:

[25]**1**. If $F : \mathbf{R}^n \to \mathbf{R}^m$ is a linear transformation, prove that $F_*(v_p) = F(v)_{F(p)}$ for any $v_p \in T_p(\mathbf{R}^n)$ with $p \in \mathbf{R}^n$.

[25]**2**. Prove the following Lemma. If β is a curve with constant curvature $\kappa > 0$ and zero torsion, then β is a part of a circle of radius $\frac{1}{\kappa}$.

[25]**3**. prove that a curve is acylindrical helix if and only if its spherical image is a part of a circle.

[25]**4**. For a cylindrical frame field E_1, E_2, and E_3
(a) Find the attitude matrix, **(b)** Starting from the basic cylindrical equations $x = r\cos\theta,\ y = r\sin\theta,\ z = z$ show that the dual 1-forms are

$$\theta_1 = dr, \quad \theta_2 = rd\theta, \quad \theta_3 = dz$$

(c) For a function $f(r, \theta, z)$ show that

$$E_1[f] = \frac{\partial f}{\partial r}, \quad E_2[f] = \frac{1}{r}\frac{\partial f}{\partial \theta}, \quad E_3[f] = \frac{\partial f}{\partial z}$$

(d) Find the connection 1-forms, **(e)** Verify the Cartan's Structural equations weith this example.