MATH. 343
PARTIAL DIFFERENTIAL EQUATIONS
SECOND MIDTERM EXAM

April 25 , 2002 Thursday 08.40-10.30, SAZ-20

QUESTIONS:

[40]**1**. Let $u(x,y)$ satisfy the equation

$$u_{xx}-2u_{xy}+u_{yy}+2u_x-2u_y+4=0$$

in a region of the $xy-$plane.
(a) Classify the equation,
(b) find its characteristics,
(c) find the complete integral,
(d) find the solution when $u=2$, $u_y=0$ on the line $y=0$,
(e) find the solution when $u=2$, $u_x=0$ on the line $x+y=0$,
(f) interpret your results in (d) and (e).

[30]**2**. (a) Prove that an interior Dirichlet problem of the Poisson equation

$$\nabla^2 u=F \text{ in } D, \quad u=f \text{ on the boundary } \quad \partial D \text{ of } D$$

where F and f are continuous functions in D and on ∂D respectively, has a unique solution. We assume that D is closed and bounded. (We assume the existence of the solution)
(b) Prove that the two solutions of a Neumann problem of the the Poisson equation differ by a constant.
(c) Let $\bar{D}$ be a set of points in the plane such that $x^2+y^2\leq 1$. Let $u(x,y)=x^3-3xy^2$. Find the maximum and minimum values of u in $\bar{D}$.
(d) Discuss the meaning of your result in part (c).

[30]**3**. Solve

$$xu_{xx}-(x+y)u_{xy}+yu_{yy}=\frac{x+y}{x-y}(u_x-u_y)$$